TECHNISCHE UNIVERSITÄT DARMSTADT

Didactical perspectives on mathematics and its philosophical implications

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Content

- Some remarks
- Calculating with stones
  - The personal dimension of mathematics
  - The cultural dimension of mathematics
- Philosophy and didactics of mathematics
  - Social Constructivism and traditional philosophy
  - Didactical perspectives on …

Calculating with stones – an example case

Task:

Please perform the following calculations using your material. For which aspects is your material helpful, less helpful or even useless?

573 + 56 =

327 – 48 =

18 : 5 =

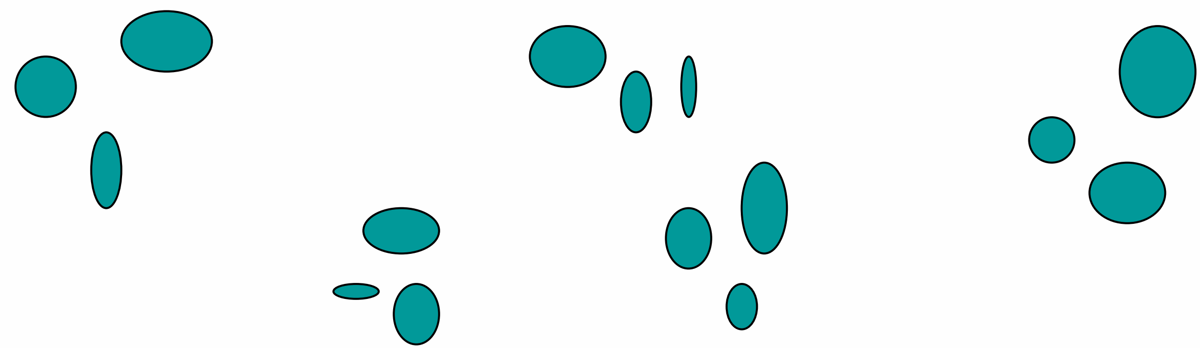
For each calculation, please describe your approach.

A learning environment – materials of visualization

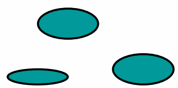
- peas
- stones (in different sizes)
- clay tablets
- laces of the Inca
- finger bargaining
- Roman numbers
- abacus

A Classroom Conversation

‚We don‘t have enough stones. They are only sufficient to carry out the division.'



Antonia pointed at the table showing a distribution of 18 stones to 5 clusters (18 : 5 = 3 remainder 3).

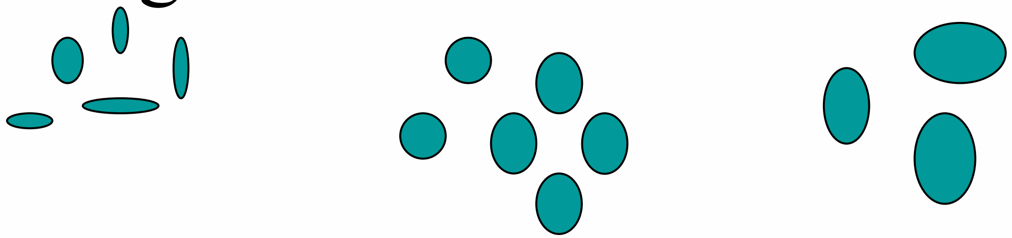


Limited Scope – Comparative Reflection

‘The group working with peas have much more in numbers, more than enough.’

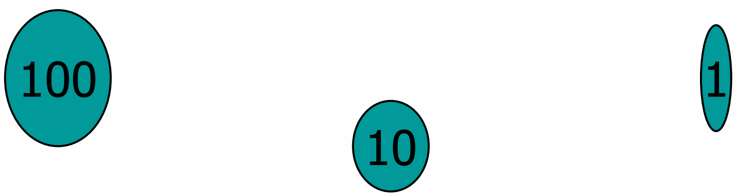
What makes the pebbles different from the peas ?

Valentine started to sort the pebbles according to size.



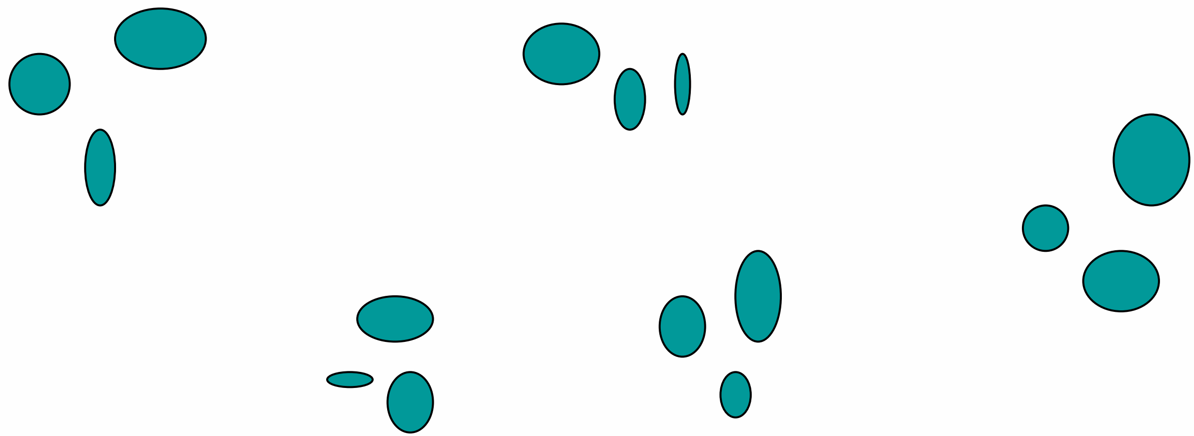
On the way to a mathematical breakthrough

‘A pebble has to represent more than one, otherwise there are not enough.’

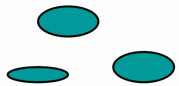


Valentine labels the stones hesitantly.

A good idea

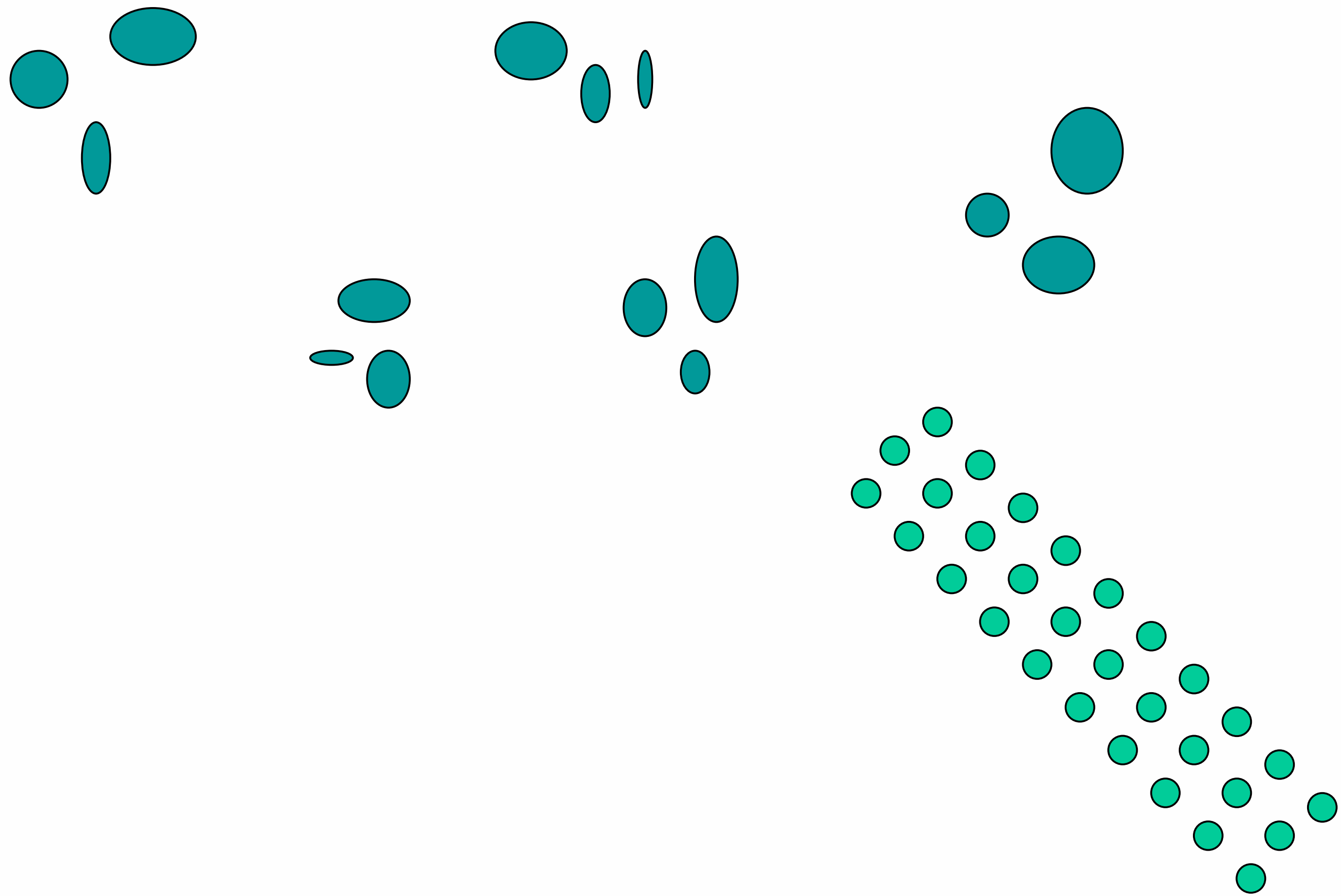


18 : 5 = 3 (remainder 3)

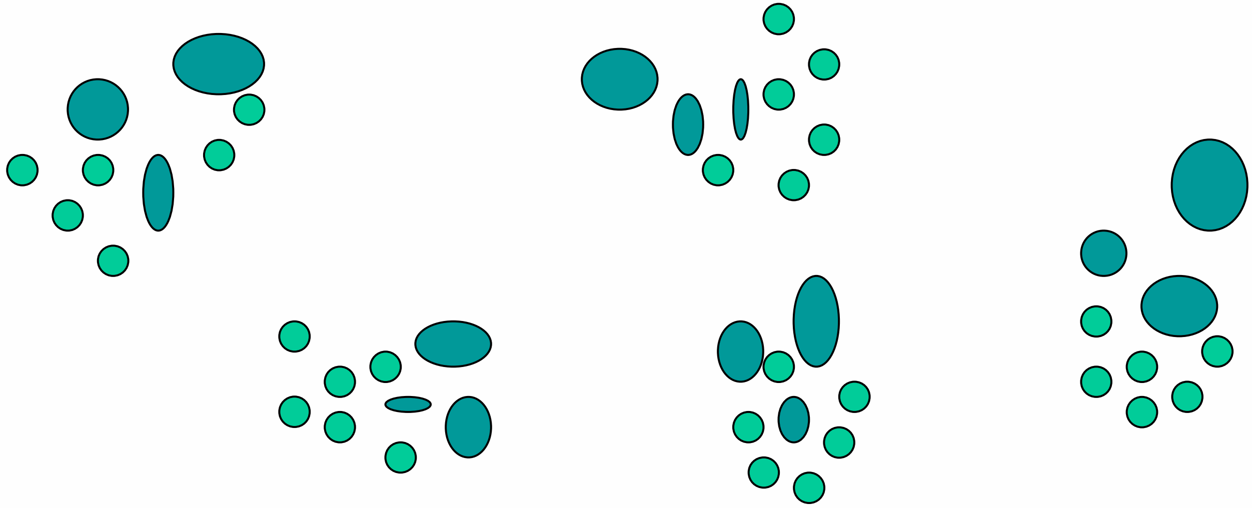


Antonia replaces 3 pebbles by 30 peas

Replacement



A pea represents a tenth



The Personal Dimension of Mathematics

- Mathematics makes sense, personally
  - Come in action
  - Connect mathematical concepts with lifeworld concepts

- Mathematics is human, not pure
  - Mathematical values
  - Context bound

Mathematics is learnable and has a meaning

Dialogical Learning of Mathematics (Gallin /Ruf 1998)

Singular ⇔ Regular

Central Ideas (Kernideen)

Preview ⇔ Review

The Cultural Dimension of Mathematics

- Function in a specific context
- Mathematical outlook – Mathematics is a cultural orientation system (Prediger 2004)
  - ➢Mathematical archaeology (Skovsmose 1998)
  - ➢Mathemacy (Skovsmose)
- How did the mathematical perspective emerge?

Mathematics – a cultural practice

- Basic activities of mathematics (Ethnomathematics, e.g. Bishop 1991)
  - Counting, Locating, Measuring, Planning, Playing, Reasoning
- Practical activities in arithmetic (Kitcher 1984)
  - Collecting, Combining, Separating, Correlating and Measuring
- Humanist Mathematics (Hersh 1997)

Social Constructivism vs. Absolutism and Platonism

- Epistemological and ontological assumptions of Absolutism and Platonism are rejected by Social Constructivism

(1) Mathematical knowledge is absolutely secure and objective, the cornerstone of all rationality (Absolutism)

(2) Mathematical objects all exist in some objective superhuman realm (Platonism)

- It is emphasized in the centrality of mathematical practice and social processes

Central positions of Social Philosophies

- Mathematics is seen as the outcome of social processes (history, research practice, learning practice gain in philosophical significance)
- Mathematics is fallible and eternally open to revision (reflection and handling of mistakes)
- Mathematics is context-bound and value-laden (social aspects of mathematics, critical attitude)

Didactical Principles of Social Constructivism

Respect for the learner's meanings and prior knowledge

- sensitive handling of language

Building on child-methods through learning

- Explicit-making of decisions within the development of mathematics

Inseparability of mathematics and applications

- historical origins of mathematical concepts and the problems they serve, current uses and contexts of use.

Didactical consequences of other philosophical standpoints

Absolutism

mathematical knowledge is timeless, superhuman, ahistorical

- teaching is a matter of efficient transmission

Progressive Absolutism

Learners construct knowledge, but the knowledge itself is superhuman and pre-existing

- teaching is a constructive activity with the teacher as a corrector

Didactical Theories corresponding to Social Constructivism

- Dialogical Learning
  - Constructivist (from singular to regular conceptions)
  - Not necessarily social, but intersubjective
- Ethnomathematics
  - Culturally embedded
  - Constructivist, mathematics as an activity

Didactical Theories corresponding to Social Constructivism

- Critical Mathematics Education (CME)
  - Mathematics as an outlook on the real world
  - Learning in social practice
  - Centrality of reflection at learning
  - operative capabilities less central, new technologies
- Realistic Mathematics Education (RME) – Freudenthal Institute
  - Real situations
  - Progressive mathematization (constructive)
  - Learning trajectory is pre-planned, pre-existing knowledge?

Didactical Theories corresponding to Social Constructivism

- Conceptual change, didactical reconstruction, reflective learning
  - Starting from the learners prior knowledge and conceptions
  - Initiating conceptual changes
  - Reconstructing mathematics from the viewpoint of the learner
  - Reflecting the mathematical outlook on the world

How to deal with …

- Reasoning and mathematical proofs?
- Axiomatization and axioms?
- Diagrammatic and formal deduction?
- Metaphors and theorems?
- Singular and regular conceptions?
- New technology and operative capabilities?

Reasoning and mathematical proofs

- Proof is an accepted practice of mathematicians
  - Aim is to ensure mathematical results (know that)

- Reasoning may has different aims
  - Looking for reasons (know why)
  - Unfolding connections
  - Gaining evidence
  - …

Three wishes for change

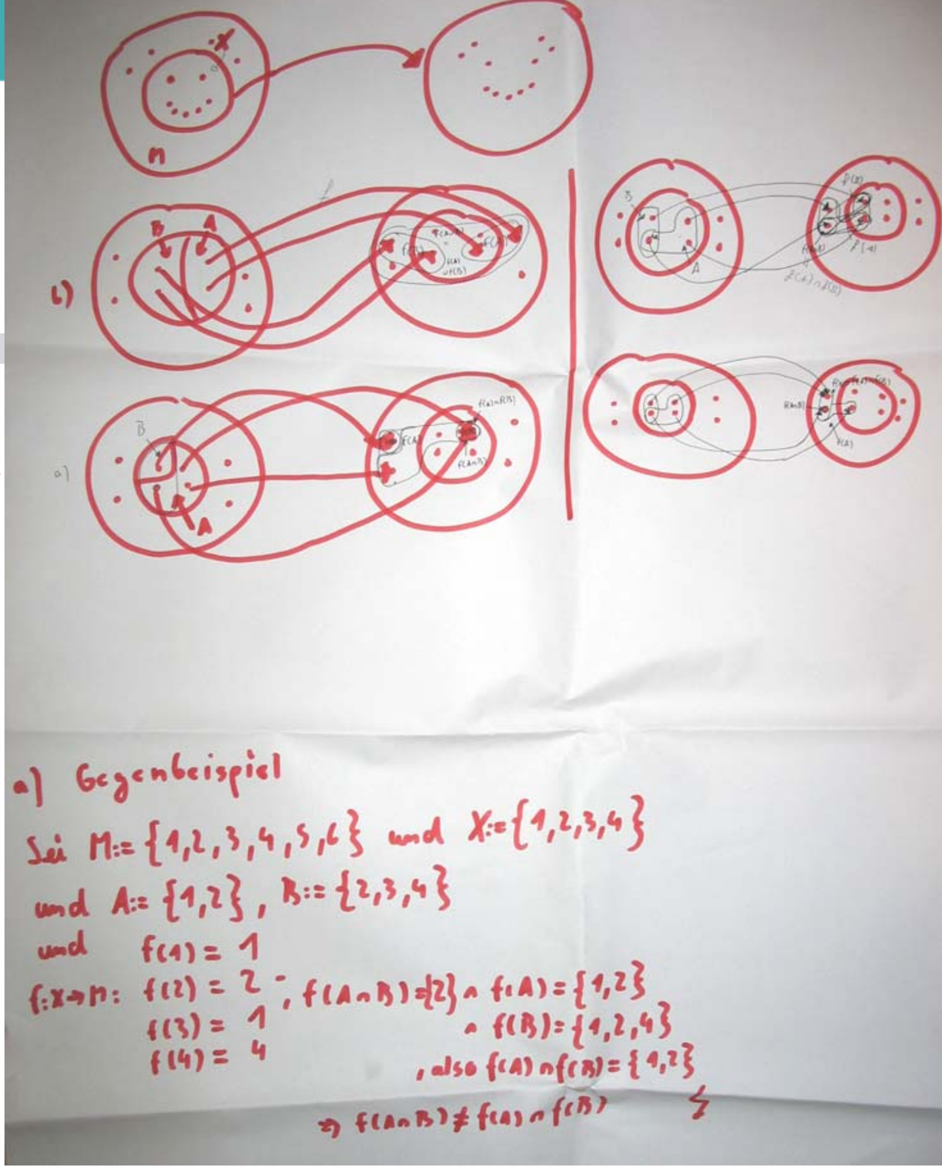
- Replace the question:
  - What is the nature of mathematical knowledge? by :
  - How does mathematical knowledge emerge (historically, scientifically, at the learning)?
- What can be seen by the mathematical outlook on the world?
- Which meaning has mathematics for humans, which should it have in the future?

Mathematics in progress

For A, B subsets of a set M. Prove or give a counterexample:

a) $f(A \cap B) = f(A) \cap f(B)$

b) $f(A \cup B) = f(A) \cup f(B)$



Thank you for your attention!

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