

Why and how to write a history of higher-dimensional algebra

David Corfield, Max Planck Institute, Tübingen,

New blog: [The n-category café](#)

Notes for a talk given to the 'Towards a New Epistemology of Mathematics' Workshop, Free University, Berlin, 14-16 September 2006.

In the time that I have I shall have to suppose much of the argument of my [paper](#) which bring Alasdair MacIntyre to the philosophy of mathematics. MacIntyre is a moral philosopher famous for his return to Aristotle and Aquinas.

...if the Thomist is faithful to the intentions of Aristotle and Aquinas, he or she will not be engaged, except perhaps incidently, in an epistemological enterprise... (p. 148)

(Page numbers refer to 'The Tasks of Philosophy' (CUP, 2006) unless otherwise specified)

This might appear to put me add odds with the direction of this workshop. But let me clarify that by this MacIntyre means that the Thomist is not engaged on an enterprise which seeks to found his or her own claims to knowledge. The Thomist is not a Cartesian. A theory of enquiry is a third-person affair about participation in a community of enquirers. Belonging to such a community entails that

It is more rational to accept one theory or paradigm and to reject its predecessor when the later theory or paradigm provides a stand-point from which the acceptance, the lifestory, and the rejection of the previous theory or paradigm can be recounted in more intelligible historical narrative than previously. An understanding of the concept of the superiority of one physical theory to another requires a prior understanding of the concept of the superiority of one historical narrative to another. The theory of scientific rationality has to be embedded in a philosophy of history. (1977, 467)

Now, the observant among you will notice in my title an echo of André Weil's 'History of Mathematics: Why and How?' which Leo [Corry](#) considered at a conference 'Mathematics and Narrative' in Mykonos.

The history of science has come a long way since the triumphalist accounts of steady progress overcoming limitations of the past. These were largely about individual white male heros, and full of anachronisms and inaccuracies. The message was that triumphs ought to be quickly recognised by any competent person. There was nothing about the social except as that which gets in the way of the rational individual. The truth is the cause of our discoveries. Corry quotes Elkana:

The conviction emerged and grew, leading up to its positivistic absoluteness in the Victorian frame of mind, that not only there is one reality with it immutable laws, but also that we humans are on a sure course to find out all, or at least cumulatively more and more about the reality: one nature, one truth about nature. Science, the chief glory of Western culture since the scientific revolution, is an inevitable unfolding of knowledge; what we know we had to know -- if not here, then there, if not now, then at another time, if not discovered by one man,

then by another.

Yehuda Elkana, "The Myth of Symplicity", in Gerald Holton and Yehuda Elkana (eds.), *Albert Einstein. Historical and Cultural Perspectives*, Princeton, Princeton University Press (1982), pp. 205-251, on pp. 205-206.

This characterisation is very similar to what MacIntyre calls Encyclopaedist.

Against this Shapin and Shaffer's *Leviathan and Air Pump* boldly took on the greatest Encyclopaedist claim, that the very method of science itself was part and parcel of Europe freeing itself from dogma. Two models of science on offer. Boyle's experimental and Hobbes' deductive. The political situation decided in favour of Boyle.

In this type of history, it's not the ways things are which causes triumph. Rather the triumphant constitute the way things are. History then shifts to look at the excluded - lab technicians, editors, women, exterior social. First port of call in understanding any decision making is the interests of the parties.

For MacIntyre, this version of enquiry is *genealogical*, deriving from a Nietzschean perspective:

Truth is "A mobile army of metaphors, metonymies, anthropomorphisms, a sum, in short, of human relationships which, rhetorically and poetically intensified, ornamented and transformed, come to be thought of, after long usage by a people, as fixed, bunding, and canonical. Truths are illusions which we have forgotten are illusions, worn-out metaphors now impotent to stir the senses, coins which have lost their faces and are considered now as metal rather than currency?" (Über Wahrheit und Lüge im aussermoralischen Sinne I)

The genealogist's tasks involve the discrediting of canons, by the unmasking of the will to power.

We now have two versions of enquiry, Genealogist and Encyclopaedist, and they agree: "Either reason is thus impersonal, universal, and disinterested or it is the unwitting representative of particular interests, masking the drive to power by its false pretensions to neutrality and disinterestedness." (Macintyre 1990a:59)

MacIntyre was very struck in the 1970s by debates between Kuhn, Lakatos, and Feyerabend about how to write history. This tradition dies out. Today any philosopher of X using cameos of X to prove a point is easily picked off by a genealogist historian.

I came to MacIntyre as someone who had trodden a line between the irrationalism of Kuhn, unsuccessfully distancing himself from the Genealogical side, and the rewriting of history of Lakatos, 'rational reconstruction', in his quest for timeless programme independent criteria for progress, i.e., someone erring on the Encyclopaedist side.

But there is a third possibility. This MacIntyre calls tradition-constituted enquiry.

"...just because at any particular moment the rationality of a craft is justified by

its history so far, which has made it what it is in that specific time, place, and set of historical circumstances, such rationality is inseparable from the tradition through which it was achieved. To share in the rationality of a craft requires sharing in the contingencies of its history, understanding its story as one's own, and finding a place for oneself as a character in the enacted dramatic narrative which is that story so far. The participant in a craft is rational qua participant insofar as he or she conforms to the best standards of reason discovered so far, and the rationality in which he or she thus shares is always, therefore, unlike the rationality of the encyclopaedic mode, understood as a historically situated rationality, even if one which aims at a timeless formulation of its own standards which would be their final and perfected form through a series of successive reformulations, past and yet to come.” (MacIntyre 1990a: 65)

Histories of intellectual enquiry naturally reflect conceptions of such enquiry. Obvious targets for historians are the doxologists, or extreme Whigs, but we must be careful not to conflate the Encyclopaedist and the Thomist:

"The narrative structure of the encyclopaedia is one dictated by belief in the progress of reason...Narrative of the encyclopadist issues in a denigration of the past and an appeal to principles purportedly timeless...So the encyclopaedists' narrative reduces the past to a mere prologue to the rational present." (Macintyre 1990a: 78)

"For the genealogist this appeal to timeless rational principles has, as we have seen, the function of concealing the burden of a past which has not in fact been discarded at all." (*ibid.*)

They must write a history of that to be undermined, and a history of how to avoid the pitfalls of the taken-for-granted.

"The Thomists' narrative...treats the past...as that from which we have to learn if we are to identify and move towards our telos more adequately and that which we have to put to the question if we are to know which questions we ourselves should next formulate and attempt to answer, both theoretically and practically." (Macintyre 1990a: 79)

What we're after is history written retrospectively without the excesses of Whiggism, i.e., its self-justification without proper self-examination. History should be used to expose one's partialities:

"Despite strictures about the flaws of Whig history, the principal purpose for which a mathematician pursues the history of his subject is inevitably to acquire a fresh perception of the basic themes, as direct and immediate as possible, freed of the overlay of succeeding elaborations, of the original insights as well as an understanding of the source of the original difficulties. His notion of basic will certainly reflect his own, and therefore contemporary, concerns." (Langlands: 5)

We can confront the past not to seek a confirmation of our present narrative, but to 'falsify' it, or better to challenge the 'naturalness' of contemporary ways of viewing a problem. So a narrative must be truthful, needs to aim at the truth. It needs to use the

past to explain how partial viewpoints were overcome, and to discover whether we have acquired new partialities, and have failed to learn from our predecessors. When we come to read such accounts there should be no 'suspension of disbelief', as Leo suggests we do in mathematical fiction.

Take a movement like n-category theory. Its central message is to replace equation by isomorphism, isomorphism by equivalence. It has an inbuilt thrust to understand the past as lower steps of a ladder with occasional glimpses of higher things. John Baez has a sketch of the [history](#) of n-categorical physics, going back to Maxwell in 1876, which at this early stage might be described as a list of achievements. If we try to give a narrative history of n-categories as pure mathematics, inevitably we will include close to the beginning some account of Eilenberg and Mac Lane, 60 years ago. But then much inspiration for them comes from earlier German algebra, 70 years. And where to stop, why not back to Klein and the Erlanger Program, 130 years ago? (John and I are carrying out a [categorification](#) of the Erlanger Program.) If I were funded to work seriously on this, a close study of a true history of the original would I'm sure be useful.

"... the history of all successful enquiry is and cannot but be written retrospectively; the history of physics, for example, is the history of what contributed to the making in the end of quantum mechanics, relativistic theory, and modern astrophysics. A tradition of enquiry characteristically bears within itself an always open to revision history of itself in which the past is characterized and recharacterized in terms of developing evaluations of the relationship of the various parts of that past to the achievements of the present." (Macintyre 1990a: 150)

Your history is of how things came to look, but also a history of the history of how things came to look.

But it's not just dispassionate history. You care about how things turn out. Writing of this history must be partly a rallying affair.

Is this enough to bring us to a two-state solution such as Ivor Grattan-Guinness suggests? 'Heritage' for the mathematicians' stories, 'history' reserved for the historians. Yes, if historians choose to be genealogical, but then this is a clash of philosophies not a peaceful coexistence.

Corry says:

The "thing that has been", which is the singular, the idiosyncratic, is the object of historical research, and the historian should strive to understand and convey it in her research. The "thing that might be", while of "more philosophical and of graver import", is none of the historian's professional business.

And compares the mathematicians' story to the historians:

We know what will happen: drama arises because we know that it will happen.

vs

**"Things can happen this way, but they can also happen in a quite different way"
(Walter Benjamin)**

But as soon as their aim is to convey that always "things could have gone just as 'well' but differently" this is philosophical. If a historian is rejecting the idea that mathematical decisions may be good, and further may be made for good reasons, then this is a philosophical position, and one which clashes with the philosophy underlying the mathematicians' accounts. Of course, I'm not suggesting that all historians believe this.

If they believe in a *telos* the mathematician must believe that some decisions were good, which doesn't mean they had to be taken, but that it was good that they were taken. And they can't all have been down to good luck. If the historian has no problem with the notion that a decision can be made for good reasons, what is the need to separate. Each can complement the other.

A history of something like higher-dimensional algebra can't make an individual or an institution or the national community of some mathematicians the central star of the show. What binds the historical events together is an ever-changing continuity, in which goals become more explicit and modified.

There's a danger then that something of the truth of mathematics will be lost if no large-scale dramatic narratives such as these are written. How can we put any fence around a collection of pieces of mathematical activity and claim it hangs together. Individual, community. How long can a community persist? Leo gives 35 years of algebra, what connects the beginning to the end?

Let's turn from MacIntyre to David Carr, a phenomenologist. In *Time, Narrative, and History*, Indiana University Press, Bloomington, 1986, he argues that we don't just live an unmediated experience which we give a narrative gloss to, rather we already experience it as narrativised.

He goes on to compare the historical narratives of the historian and the practitioner:

...the narrative structure and narrational activity within communal existence is, as we have insisted, primarily practical in character; historical narrative, by contrast is cognitive and seeks an objective representation. The former is engaged in action and has an interest in its outcome; the latter is detached and disinterested, and aims only at truth. The second difference concerns the temporal standpoints of the narrators in each case. Our "practical" narrator is situated in medias res, whereas the historical narrator looks back at actions and events already completed. That gives the latter the well-known (and already discussed) advantage of hindsight over his subjects: he knows how things turned out, knows the difference between the intended consequences and the real consequences of their action, etc.

These differences between narrative agent or participant and narrative historian are operative and important: there is no denying the importance of temporal standpoint and of the difference in attitude (engaged or detached) in relation to a lived or performed sequence of human events. At the same time we should like to

emphasize several respects in which these differences are mitigated. And we shall do this not by denying objectivity and hindsight to historical inquiry, but by attributing them to narrative-historical existence.

We have already pointed out ..., with respect to individual action and experience, that the narrativization that goes on there cannot be indifferent to truth where the past is concerned. Indeed, where the issue is not merely the shaping of an open future but the coherence of future, present, and past, it is important to be clear on what really happened; the past may be variously interpreted but it cannot be wished away or forcibly altered by an inventive narrative imagination. So much of one's present capacities are in continuity with, and sometimes result from, past choices and experiences that getting straight one's past can be seem as a desideratum and even a necessary condition for a coherent life. This is, of course, one of the insights on which much psychotherapy is based, as we pointed out.

We have this scheme:

Histories	
Practitioners'	Historians'
practical	cognitive, objective
engaged	detached
interest in outcome	disinterested
in medias res	know outcomes

Important differences, but differences are mitigated

objectivity (truth is important)
hindsight (thinking in a future perfect tense)

Carr continues:

A concern for the truth of the past plays the same role in the case of the community. Members often debate the facts of the past, precisely because they are so important in the constitution of the present and the future. This is not to deny that the past is often manipulated, especially where social story-telling is political and persuasive in character. The personal past is often distorted too, deliberately or not. My point is merely that a genuine interest in the truth of the past is compatible with and indeed important for the practical narrative constitution of communal existence. Equally, objectively-oriented historical enquiry and research are not disqualified from playing a role in the ongoing political and social debates of a community; on the contrary, they can and do contribute to them.

We are not commenting here, it should be noted, on the success with which truthfulness about the past is actually attained. Our point concerns the interest in or commitment to truth, and we are only saying that this is not restricted to history as a discipline. It is true that the discipline has among other things developed techniques for discovering and evaluating evidence in order to implement its commitment to truth. A justified suspicion that partisanship in the events of the day can distort our view of the past has led to the emphasis on

detachment and objectivity. But these in turn, once achieved, can be put in the service of engagement in the present and the shaping of the future.

This raises the question “Who are historians writing for?”.

As for the hindsight which is characteristic of historical enquiry, this too is not exclusive to the latter, at least not formally. Socially constitutive narrative, like the narrative structure of individual life and action, has a prospective-retrospective form. In anticipating the future, it aims at, and largely achieves, that quasi-hindsight that we characterized earlier, borrowing Schultz's term, as the future perfect. Far from waiting passively for things to happen, communities negotiate with the future and understand the present in the light of that future.
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Carr's position would require historians and mathematicians to be brought into a much closer relationship, and rightly so:

Far from dealing with past events which are fixed and whose consequences are clear, historians here deal with events whose consequences are still being felt and are operative in the present. 173

David Carr *Time, Narrative, and History* 1986 Indiana University Press Bloomington

There's a role for the mathematician helping the historian make sense of the thought of a mathematician of say 1890. If you just avail yourself of what is published until then, you will have to have an extraordinary ability to be able to say what that mathematician was thinking, had just begun to glimpse, especially as what emerges afterwards is often complicated and not the way you'd learn the subject, which is later tidied up.

And there's a role for the historian in disrupting some of the current understandings.

Finally, with regard to ourselves, speaking of a community, cohesive linked, one which want a better philosophy of mathematics, we'd better not end up as a bunch of fragmentary contributions, enjoying our distance to the mainstream, or perceiving ourselves as an annexe. Let's establish the canonical texts of our movement.

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