Concepts of knowledge in mathematical practice — a cluster analysis

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(**Phi**losophy of **M**athematics – **S**ociological **A**spects and **M**athematical **P**ractice)

19^{th} October 2007

PhiMSAMP2, Utrecht

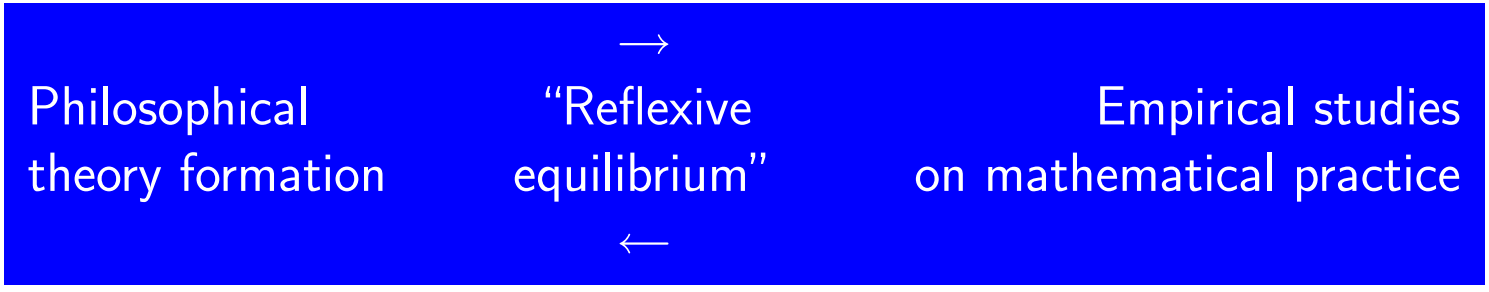
Outline of this talk

- What's the (philosophical) issue?
- A socio-empirical study
- Cluster analysis of the data
- Philosophical interpretation
- Open questions

What's the (philosophical) issue?

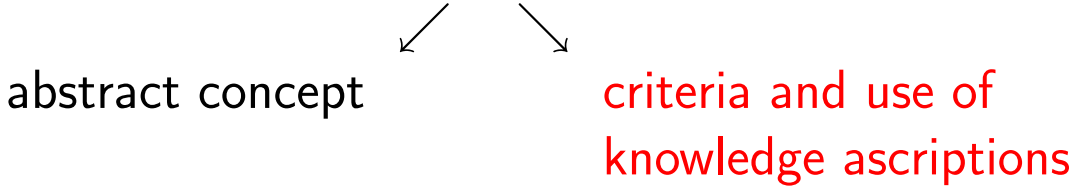
- **Aim:**

  A philosophical **theory of mathematical knowledge** that fits mathematical practice.



- **First step:**

  Conduct an empirical study on the concept of mathematical knowledge **employed by working mathematicians**.



A socio-empirical study

- **Characteristics of the project**

topic	the concept of knowledge in mathematical practice
type	quantitative web-based survey with qualitative free text part
types of questions	multiple choice, ordinal variables
runtime	1 month (August '06)
target group	international working mathematicians
participants	newsgroup readers

A socio-empirical study (cont'd)

- **The questionnaire**

  Three parts:

  ▷ Part I on personal data

  ▷ Part II on abstract concept of knowledge & proof

  ▷ Part III on knowledge ascriptions (4 scenarios)

Cluster analysis of quantitative survey data

Aim

Find clusters that can be interpreted as "**types of working mathematicians**", **corresponding to different concepts** of mathematical knowledge the representatives employ.

- Step 1
  - ▷ Choose small number of cluster variables from part III.
  - ▷ Find clusters that can be interpreted as **different concepts of mathematical knowledge**.
- Step 2
  - ▷ Take larger number of cluster variables from all parts of the questionnaire.
  - ▷ Sharpen cluster interpretation, try to identify corresponding "**types of working mathematicians**".

The concept of knowledge in mathematical practice

After his Ph.D., John continues his mathematical career. Five years after the paper was published, he listens to a talk on anti-Jones functions. That evening, he discovers that based on these functions, one can construct a counterexample to the Jones conjecture. He is shocked, and so is professor Jones.

Does John know that the Jones conjecture is false?

○ yes ○ almost surely yes ○ almost surely no ○ no ○ can't tell

Did John know that the Jones conjecture was true on the morning before the talk?

○ yes ○ almost surely yes ○ almost surely no ○ no ○ can't tell

Next Reset

Cluster analysis of quantitative survey data (cont'd)

Step 1 — methodological remarks

- The choice of cluster variables is theory guided!
- 3 cluster variables from a key scene in Scenario 1:

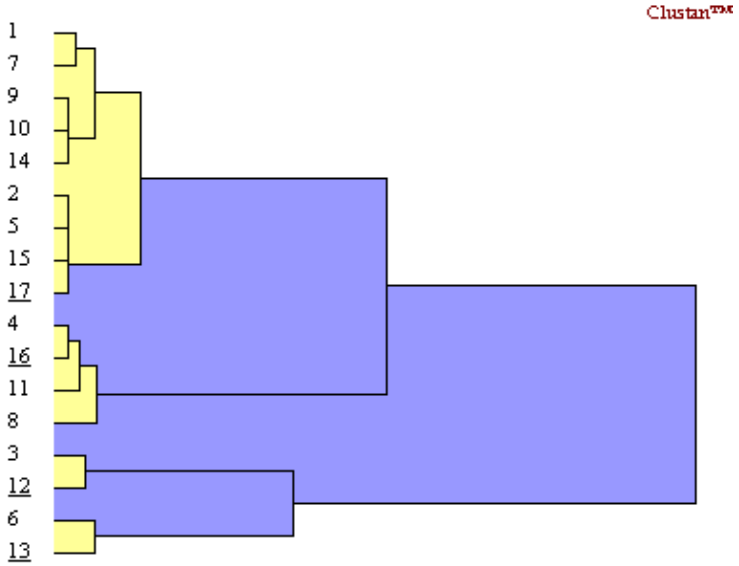
QJones6 „Does John know that JC is **true**?“ asked **before** talk.

QJones7 „Does John know that JC is **false**?“ asked **after** talk.

QJones8 „Did John know that JC was **true** on the morning before the talk?“ asked **after** talk.

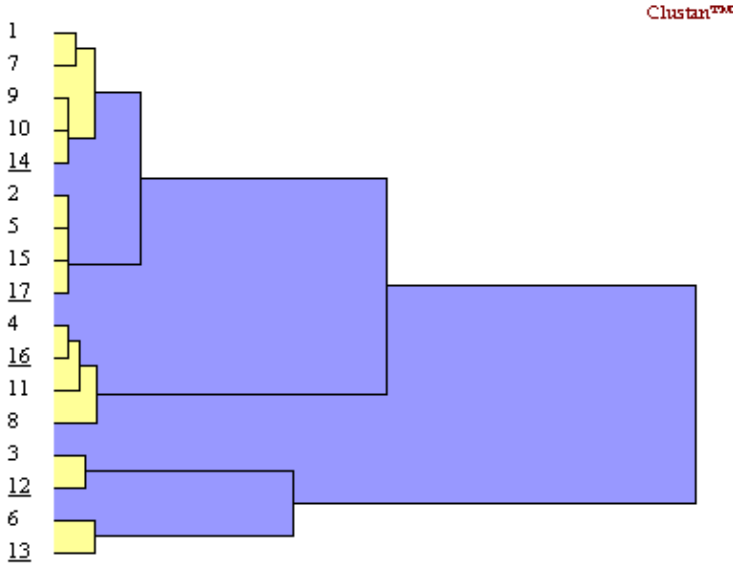
- Preparation of the data
  - ▷ Pre-sorting after country – here: subgroup “Germany” (60 $\searrow$ 17 cases, but marginal influence on structure of results).
  - ▷ Ordinal variables (5 parameter values) $\longrightarrow$ interval variables
  - ▷ Eliminate listwise: missing data, ‘can't tell’ cases, 1 “outsider”

Cluster analysis – results



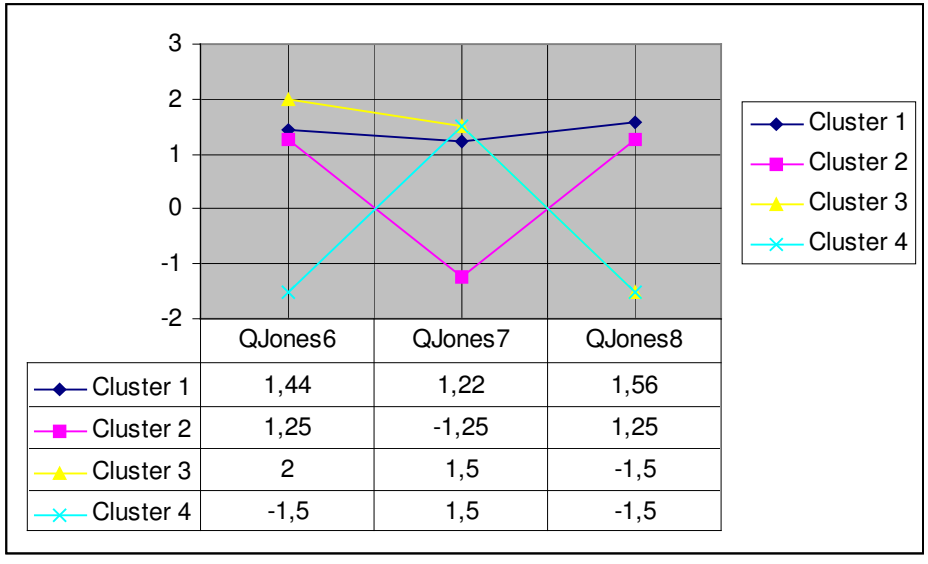
4 cluster solution

Cluster analysis – results (cont'd)



5 cluster solution

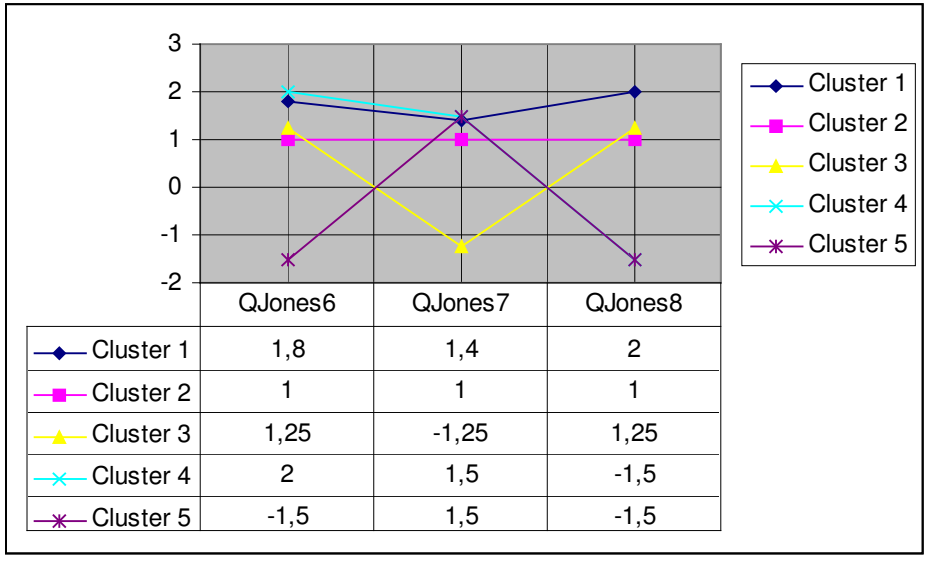
Cluster analysis – results (cont'd)



	QJones6	QJones7	QJones8
Cluster 1	1,44	1,22	1,56
Cluster 2	1,25	-1,25	1,25
Cluster 3	2	1,5	-1,5
Cluster 4	-1,5	1,5	-1,5

Line Plot of 4 cluster solution

Cluster analysis – results (cont'd)



	QJones6	QJones7	QJones8
Cluster 1	1,8	1,4	2
Cluster 2	1	1	1
Cluster 3	1,25	-1,25	1,25
Cluster 4	2	1,5	-1,5
Cluster 5	-1,5	1,5	-1,5

Line Plot of 5 cluster solution

Philosophical interpretation of step 1 clusters

- **Aim of step 1:**

  Find clusters that can be interpreted as **different concepts of mathematical** knowledge.

- **Claim:**

  Some of the clusters from step 1 correspond to different philosophical theories of knowledge

  ▶ points to **different concepts knowledge** employed.

- **Suggestion:**
  (regarding the 5 cluster solution):

  Cluster 1 (& Cluster 2) $\leftrightarrow$ Contextualism

  Cluster 4 $\leftrightarrow$ classical Invariantism

Outlook & open questions

- **Step 2**
  - ▷ Will clusters from step 1 be reproduced in step 2?
  - ▷ Will step 2 suffice to identify corresponding "**types of working mathematicians**" – e.g. in terms of working habits?
- **Further studies**
  - ▷ Cluster specific studies.
  - ▷ Use survey study as a learning study.
- **Philosophical lessons to learn**
  - ▷ Is there an "overall" epistemology of mathematics?
  - ▷ On which notion of proof could it be based?

Thanks for your attention!