Ethnomathematics and the philosophy of mathematics (education)

Karen François and Bart Van Kerkhove*

Centre for Logic and Philosophy of Science, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, Belgium

E-mail: karen.francois@vub.ac.be; bart.van.kerkhove@vub.ac.be

1 Introduction

This paper looks into the field of enquiry called ethnomathematics and the influence it has exerted on the philosophy of mathematics and mathematics education. Although a number of pointers to the most relevant and survey literature will be included, it is important to note that we shall not here be providing a systematic overview of empirical-anthropological studies in ethnomathematics, as originally conceived. Ethnomathematics is still being identified with this type of studies, although they have never gone under its name, being included instead in several domains, such as anthropology or history. Nowadays, the bulk of this research falls under the label of cognitive studies (cf., e.g., De Cruz et al., 2010, in this volume). What the reader may instead expect here is a general survey of theoretical work done within the research discipline of ethnomathematics during the last few decades, with a particular focus on Western science and society.

In Section 2, we first take a broad view, estimating the seemingly paradoxical place mathematics occupies in modern society, being as ubiquitous and important as it is invisible and loathed. In Section 3, we first elaborate on the shifted meaning of the concept 'ethnomathematics', and then explore a number of domains in which it has been applied. Until the early 1980s, only mathematical practices of 'nonliterate' peoples were studied, in an attempt to show that their mathematical ideas were as sophisticated as the modern, 'Western' ones. Then a broadening of the 'ethno' concept was proposed, to include all culturally identifiable groups. As a result, today, within the ethnomathematics discipline, scientists are collecting empirical data about the mathematical practices of culturally differentiated groups, literate or not. 'Ethno' should thus no longer be understood as referring to the exotic.

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This changed and enriched meaning of the concept 'ethnomathematics' has had an impact on both the philosophy of mathematics and mathematics education. Within the philosophy of mathematics, which we discuss in Section 4, it has lent weight to studies of mathematical practice as alternative or complementary to foundational studies. Within the field of mathematics education, ethnomathematics gained a more prominent role, since it became meaningful and relevant to explore various aspects of mathematical literacy in the context of Western curricula. In Section 5, we discuss a number of possibilities and dangers this has opened, and on the basis of this present ethnomathematics as an alternative, implicit philosophy of professional and school mathematical practices.

2 Mathematics in society

Mathematics pervades our everyday lives, sometimes obviously and sometimes on a more hidden or implicit level. The rate at which we have to, or at least are expected to, process numerical data is indeed stupefying.[1] But of course this superficial appearance is only a symptom of something deeply structural: through the ever growing impact of science and technology, our entire society has become thoroughly 'mathematized' (Resnikoff and Wells, 1973; Kline, 1990). Strangely enough, this evolution has brought about a rather perverse effect, as reported by Morris Kline:

> Just as a phrase either looses meaning or acquires an unintended meaning when removed from its context, so mathematics detached from its rich intellectual setting in the culture of our civilization and reduced to a series of techniques has been grossly distorted. Since the layman makes very little use of technical mathematics, he has objected to the naked and dry material usually presented. Consequently, a subject that is basic, vital, and elevating is neglected and even scorned by otherwise highly educated people. Indeed, ignorance of mathematics has attained the status of social grace. (Kline, 1990, p. 16)

In other words, the more we have become dependent upon mathematics, mostly in an indirect or invisible way, the less we are actually understanding its principles. While one needs to know little about the inside operations of a car or a personal computer in order to use them effectively, we are not only referring here to sophisticated applied mathematics, but also to basic skills, such as elementary probability theory. John Allen Paulos comments:

[1] E.g., Butterworth (1999) has estimated: "At a very, very rough guess, I would say that I process about 1,000 numbers an hour, about 16,000 numbers per waking day, nearly 6 million a year. People whose job entails working with numbers, in supermarkets, banks, betting shops, schools, dealing rooms, will process many more than this" (p. x).

> I'm distressed by a society which depends so completely on mathematics and science and yet seems so indifferent to the innumeracy and scientific illiteracy of so many of its citizens. [...] I'm pained as well at the sham romanticism inherent in the trite phrase 'coldly rational' [...] and at the belief that mathematics is an esoteric discipline with little relation or connection to the 'real' world. (Paulos, 1990, p. 134)

Despite there being arguably deeper reasons for perceiving mathematics as an extremely difficult, unworldly and thus unsympathetic subject (see below), this 'distorted picture' has often been identified with failing educational policy.[2] As performance in mathematics is, on the whole, fairly poor in comparison with other subjects, it is widely argued that something must be fundamentally wrong with its teaching. Morris Kline for example, in connection with his previous remark about our loss of affinity for the 'backbone' of our civilization, made this link:[3]

> School courses and books have presented 'mathematics' as a series of apparently meaningless technical procedures. Such material is as representative of the subject as an account of the name, position, and function of every bone in the human skeleton is representative of the living, thinking, and emotional being called man. Just as a phrase loses meaning or acquires an unintended meaning when removed from its context, so mathematics detached from its rich intellectual setting in the culture of our civilization and reduced to a series of techniques has been grossly distorted. (Kline, 1990, p. 15–16).

Although this is an apt criticism, it cannot be the entire story. To begin with, feedback mechanisms are in place between general performance on the one hand, and the quantitative and qualitative supply of teachers (and expert policy makers) on the other, so that it is a tricky affair to pinpoint at which of these levels (most of) the problems start or are reinforced. International studies have indeed shown both that the level of recruitment of math teachers is well below target and that the enrolment of students in higher math education has been steadily declining over the past decades. This could be partly due to a poor public image of mathematics, circumstantial evidence shows. First, there are the cultural complaints ventilated by Kline and Paulos above. Second, teachers prefer to lecture on other subjects, with the consequence that similar staff problems are less common in these other areas (Sam, 1999, p. 19–20).

What about the deeper causes of these unfortunate trends? Although largely conjectural, the postmodern critical, skeptical and even hostile atti-

[2] Cf. (Sam, 1999, p. 21).

[3] Note that Kline also wrote separate monographs about the (then) contemporary 'debacles' of elementary (Kline, 1973) and undergraduate mathematical education (Kline, 1977).

tude towards science might be a good candidate. During the 1960s, there was a growing awareness in Western society of the inherent limits and—possibly detrimental—external effects of its reigning development model: one based on capitalism, science and technology, and that recognized no limit to economical success or to the mastery of nature. The era indeed witnessed the publication of a rising number of highly critical books commenting on this situation, including *Silent Spring* (1962) by Rachel Carson (on the ecological costs of pesticides), *The Population Bomb* (1968) and *Ecocatastrophe* (1969) by Paul Ehrlich, and of course *The Limits to Growth* (1972), the famous first report of the Club of Rome, by Dennis Meadows et al. The main message for science was that it is an activity that is intimately linked with, and that cannot be cut off from, the rest of society. This theme was picked up, among others, by Jerry Ravetz in his timely *Scientific Knowledge and its Social Problems* (1971). As Sardar explains:

> If science is seen as a craft [rather than as an autonomous fact-discovering machine], then 'truth' is replaced by the idea of 'quality' in the evaluation of scientific output. Quality firmly places both the social and ethical aspects of science, as well as scientific uncertainty, on the agenda. Ravetz showed that in the overall practice of contemporary science one could identify four categories that were seriously problematic: shoddy science, entrepreneurial science (where securing grants is the name of the game), reckless science, and dirty science; and they are all involved with runaway technology. He showed further that quality in science depended largely on the morale and commitment of working scientists and was reinforced by the moral acumen of the leadership of the scientific communities (Sardar, 2000, p. 38–39)

The general wariness of science that has since established itself is confirmed when it comes to mathematics. It is perceived as unimportant, dull, coldly calculating, thus potentially threatening and impoverishing (Boyle, 2000). One might indeed ask whether mathematics, the apex as well as one of the main instruments of hard science, should not be identified with the latter's excrescences. If we are to believe Ravetz, then an important role is to be played here by mathematicians themselves. Do they care about their reputations, and what are they prepared to invest in them? Let us briefly look into two issues connected with this question: ethics and popularization.[4]

It is rather surprising how little attention has been devoted to the ethical side of mathematics. And even when it is actually attended to, people remain reluctant to get too deeply involved. E.g., Reuben Hersh, one of the leading mathematical humanists,[5] has remarked that, contrary to moral

[4]Obviously, education—one of the central themes of this paper—is another one.

[5]In view of books such as (Davis and Hersh, 1981, 1990), but also of numerous articles published to date.

considerations about *any other* discipline, for mathematics, surely these are not "intrinsic to the actual practice of the particular profession" (Hersh, 1990, p. 13). Put simply, mathematicians—as opposed to, say, chemists—are not the fathers of any artifacts that put an actual burden on, or pose a direct threat to, society or environment. Hersh continues that "it's hard to see significant ethical content in improving the value of a constant in some formula or calculating something new" (ibidem). He further holds that any moral issues to be addressed are personal (not corporate) and academic (not global) in character, and he lists "five different categories of people to whom we [mathematicians] have duties: staff, students, colleagues, administrators, and ourselves" (ibidem). Cases involved may concern discrimination, innumeracy, loyalty, fraud, etc. The only more socially embedded topic referred to, "a little out of date, but interesting" [sic] (Hersh, 1990, p. 14), is that of the affinities between mathematics and the military.

The latter is indeed one of the few (politico-)ethical issues surrounding mathematics that have been studied in any depth. There is, e.g., the pathbreaking work of historian Jens Høyrup with mathematician Bernhelm Booss-Bavnbek, showing the intimate though not essential connection, past and present, between mathematics and the military (Høyrup, 1994, Chapter 8). Although the relation is considered to be one of interdependence, particular attention is devoted to warfare as an impetus for mathematical development. One might in this respect think of the active role of mathematicians in the furtherance of code making and breaking, ballistics, positioning, spying, or 'intelligent' bombing. A particular case in point is John von Neumann, who during the Second World War participated in the development project for the A-bomb, and was one of the respected advisors of the U.S. government. In the early stages of the Cold War, he also developed game theory, having on his mind, among other things, a rational justification for tougher behaviour towards Moscow.[6] Another famous example of ideological influence on mathematical development is the considerable damage to the German mathematical community, especially its Jewish members, in the interbellum period. Under the rule of Ludwig Bieberbach's programme of *Deutsche Mathematik*, important centres of mathematical research, such as the Göttingen school, suffered greatly, or even came to a virtual end.[7]

The previous issues, from education to international affairs, are naturally connected with that of popularization. Indeed well conducted popularization can (help) adjust negative images, whether justified or not. However, this is a specialty in itself that has to cope with a number of problems and limitations. To begin with, the field is deceptively diverse, as contri-

[6]Cf., e.g., (Strathern, 2001; Macrae, 1999).

[7]Cf. (Cornwell, 2003, Chapter 16) for a general account, and (Lindner, 1980; Mehrtens, 1985) for more detailed ones.

butions vary widely, depending on what elements of the subject are taken as points of entry: esoteric or merely funny puzzles, great mathematicians of all times, specific branches or programmes, cross-cultural differences etc. (Van Bendegem, 1996, pp. 216–23). Secondly, books about 'the real stuff', i.e., (Western) mathematical science, intended for an audience from outside the discipline—whether made up of other scientists willing to peep over (sub)disciplinary walls or of interested laymen—can only screen a tiny part of the gigantic field at a time. Even those written by excellent popularizers like Keith Devlin or Ian Stewart, can only provide some general considerations or mostly (have to) end up by illuminating a number of 'accessible' topics, which—unavoidably or not—fall well short of painting anything near a 'true' picture of the goings on within the field.[8] One of the possible means of countering this trend would be to devote more attention to a specific type of (hitherto neglected) vulgarization. In it, one turns away from working into the ground the internal histories (albeit from different angles), and instead focuses on the intimate connections between mathematics and daily life, more particularly through its numerous concrete *applications*. Examples include statistics, informatics, genetics, econometrics, etc. Note that the present paper aims to contribute on various fronts to this cause of 'bringing back' mathematics to where it arose, viz. the layman.

3 Cultural foundations recognized

3.1 The notion of 'ethno'

The prefix *ethno* originally refers to races, tribes, or groups of relatives. Correspondingly, *ethnomathematics* has been associated with the mathematical practices of particular tribes or indigenous, 'primitive' peoples, as well as those of a nation and/or human race.[9] In recent times, under the impulse of an encompassing research programme, the concept has received a much broader interpretation. Before we turn to that, however, a few words on the anthropological roots.

As a field of enquiry, ethnomathematics started in the 1960s, its subject being the mathematical practices of so-called illiterate (or, more politically correct: nonliterate) peoples, holding that—despite the fact that the category 'mathematics' is a strictly Western one—mathematical *ideas* are "pan-human", and are primarily "developed within cultures" (Ascher

[8] In a recent book, Devlin has himself confirmed this sorry circumstance. If one desires "to delve further into the world of mathematics, then there are a number of excellent books you can consult, written for a general audience. Most of them, however, are either written at a much more superficial level than the present one [*The Millennium Problems*], or else are focused on specific issues in mathematics" (Devlin, 2002, p. 229).

[9] Of course the latter concept, which was notoriously used to apply biological differences as a basis for segregation, has meanwhile been discredited as a result of advanced studies in genetics.

and Ascher, 1994, p. 1545). Since then, it has also become the (implicit or explicit) mission of many within the discipline to demonstrate that the practices in question and 'our' Western ones are equally complex.

Ubiratan D'Ambrosio (to whom we shall return in extenso below) has provided a more extended version of this idea, with his aim of encompassing the whole range of cultures. Thus he presupposes "a broader concept of 'ethno', to include all culturally identifiable groups with their jargons, codes, symbols, myths, and even specific ways of reasoning and inferring" (D'Ambrosio, 1997, p. 17). Here the word 'ethno' has ceased to refer to anachronistic concepts such as racial groups, primitive peoples or illiterates, and is instead given a comprehensive meaning, pointing to any group of people who share a cultural identity.

However, given the myriad of different interpretations of the concept of culture, the latter move is not uncontroversial. In static views that employ 'culture' to discriminate among groups, characteristics attributed to respective groups of people are considered essences of their culture. But pegging humans down to essences causes them to be characterized invariably and irreversibly, a way of identification that conflicts with the practices of human interaction and mutual influence. Cultural changes can thus only be appreciated exploiting a dynamic interpretation of cultural identity. Indeed identities are not homogeneous and eternal, but rather correspond to an area of tension between permanence and alteration, where—within given contexts—room is left for psycho-social growth processes. A similar dynamic interpretation of culture fully links up with D'Ambrosio's plea for educational reform: "More attention should be paid to students and teachers as human beings, and we have to realize that mathematics—the same is true with respect to other disciplines—are epistemological systems in their socio-cultural and historical perspective and not finished and static entities of results and rules" (D'Ambrosio, 1990, p. 374). We have thus come a long way from the initial meaning to the actual interpretation of 'ethnomathematics'. Similarly, ethnomusicology has evolved to comparative musicology, and more generally ethnology to ethnography, and later to cultural anthropology. The prefix 'ethno' has indeed experienced quite an evolution in its content, up to the moment where only the original term has been preserved.

The Brazilian mathematician and educationalist Ubiratan D'Ambrosio was the first, from the late 1980s, to propose a research program for ethnomathematics, based on the following analysis of the term:

> I call *mathema* the actions of explaining and understanding in order to survive. Throughout all our own life histories and throughout the history of mankind, *technés* (or tics) of *mathema* have been developed in very different and diversified cultural environments, i.e., in the diverse *ethnos*. So, in order to satisfy the drives towards survival and

> transcendence, human beings have developed and continue to develop, in every new experience and in diverse cultural environments, their *ethno-mathema-tics.* (D'Ambrosio, 1990, p. 369)

Following this, the research interests of the newly founded discipline pertain to the development, transmission and distribution of mathematical knowledge as dynamic processes *embedded in their socio-cultural context.* An important implication is that Western mathematics is also considered as having developed (and as continuing to develop) within a particular, contextual reality, *not* detached from it. What is currently known as 'academic' mathematics, though it originated in the Mediterranean area, later expanded to Northern Europe and then to other parts of the World, becoming so to say 'universal'. Nonetheless, it is difficult to deny that the codes and techniques which were developed (such as measuring, quantifying, inferring and abstract thinking) as strategies to express and communicate reflections on space, time, classifying, comparing, are contextual in their origin. Clearly, in other regions of the world, particular circumstances have given rise to different codes and techniques resulting from different perceptions of space, time, and different ways of classifying and comparing (D'Ambrosio, 2007a, p. 30). Reference to the socio-cultural roots of mathematical practices is common in the ethnomathematics literature.[10] This relates to the pragmatic grounds for the development of mathematics: the transmission and distribution of mathematical knowledge to help people cope with day-to-day reality. In the current description of ethnomathematics as given by the International Study Group on Ethnomathematics (ISGE) the narrow (anthropological) meaning of ethnomathematics as well as its broad (socio-cultural) meaning are combined as follows (from the ISGE webpage, July 2009):

> [Ethnomathematics] is sometimes used specifically for small-scale indigenous societies, but in its broadest sense the "ethno" prefix can refer to any group—national societies, labor communities, religious traditions, professional classes, and so on. Mathematical practices include symbolic systems, spatial designs, practical construction techniques, calculation methods, measurement in time and space, specific ways of reasoning and inferring, and other cognitive and material activities which can be translated to formal mathematical representation. The ISGE strives to increase our understanding of the cultural diversity of mathematical practices, and to apply this knowledge to education and development.

In the next section, we shall identify a number of (overlapping) ethnomathematics subdomains. These are identified according to the particular

[10] Cf., e.g., (Bishop, 1997, 1988; D'Ambrosio, 1989, 1990, 2007a,b; Gerdes, 1988, 1997; Pinxten, 1991; Zaslavsky, 1973, 1985, 1989).

subject that is their main target. We shall distinguish four: anthropology, history, philosophy and education.

3.2 Research domains influenced by ethnomathematics

3.2.1 Anthropology

A first important theme for ethnomathematicians is the description of non-Western mathematical practices that, in spite of colonization, have been retained and further developed by many so-called traditional cultures. From within cultural anthropology, the focus is on the description of these practices, and of the ideas underlying them (e.g., Ascher, 1998, 2002; Pinxten, 1991; Bazin et al., 2002). A considerable share of case-studies concerns the mathematical practices performed by professional groups such as fishermen, carpenters, carpet weavers, sugarcane farmers, salesmen and vendors (Vithal and Skovsmose, 1997, p. 134). The website of the International Study Group on Ethnomathematics (ISGE) gives a good overview of the ethnomathematical studies from the anthropological point of view. Studies of mathematical practices are listed by ethnicity/geography, e.g., African mathematics, Native American mathematics, Pacific Islander mathematics, African American mathematics, Asian mathematics, Math in European culture, Latino mathematics and Middle Eastern mathematics.

This study and description can also be coupled with a critical response to the superiority of Western mathematics, in an attempt to demonstrate the equivalent complexity of the ethnomathematics practices (Joseph, 1987, 1997; Bishop, 1995; D'Ambrosio, 2007a,b). The contrasts between the various mathematical practices are substantial but so are the similarities. Alan Bishop describes six mathematical competencies that every culture requires to be able to answer questions and respond to problems arising from the environment: counting, measuring, locating, designing, playing and explaining (Bishop, 1997, 1988, 1995). These six competencies are called 'mathematics' (with lower-case m) while 'Mathematics' (upper-case M) is used to refer to the Western or European version which is known world-wide. With this universal and intercultural interpretation of the six mathematical competencies Bishop emphasizes that a mathematical practice is a cultural product. In any culture mathematics is a symbolic technology that builds relationships between a person and his or her physical and social environment (Bishop, 1988, p. 147). Like D'Ambrosio, he points out that managing time and space, defining classifications and comparing are all human mathematical practices. To communicate and reflect on these universal practices, human beings develop codes and techniques that vary because of their development within a particular context (D'Ambrosio, 2007a, p. 30).

3.2.2 History

A second theme occupying ethnomathematicians is that of mathematical transmission and development. A critical stance is involved towards received views about the history of mathematics, by particularly examining the mechanisms of what is known as *Eurocentrism* about mathematics. While originally referring to a European attitude exclusively, Eurocentrism has gradually come to stand for *any* way of thinking that considers itself superior, thus justifying its own global distribution, thereby displaying an utter disregard for or even suppression of local practices elsewhere.[11] This self-sufficient way of thinking is also displayed in the way mathematical heritage is passed down. Typically, the history of mathematics indeed is and has been described as a *Western* affair, in which Arab, Chinese or Indian contributions figure as distant and exotic 'influences' at most—if at all—and certainly not as independent, let alone alternative developments.

Even prominent contemporary historians of mathematics, like Rouse Ball (early twentieth century) or Kline (middle twentieth century) have contributed to wiping out our collective memories of anything preceding and having inspired the 'Greek miracle', as Joseph laments at the outset of his book *The crest of the peacock* (1992). Chief among the intellectual debts one finds the Egyptian and Mesopotamian civilizations, acknowledged by the Greeks themselves, various clay tablets and papyri confirm. Further, the neglect of the vital Spanish-Arab contributions during the Middle Ages has to be counted with, viz., keeping record of the dormant Greek tradition before it was finally picked up again in the Renaissance. Moreover, nothing yet has been said about the (influence of the) rich Chinese and Indian cultures. Restivo (1992, Chapters 3–6) includes nice socio-historical readings on the mathematical traditions of China (relying on the unsurpassed Needham (1959)), the Arabic-Islamic world, India, and Japan. Clearly, something more complicated than the 'simple' diffusion model, with Europe at its centre, is needed. This is what Joseph (1992) sets out to provide; the success of this alternative will however not concern us here. Fortunately, in the meantime, an increasing number of historians have been going through great pains to show that the said influences were not so marginal after all, and instead profoundly shaped mathematics as we know it (or consitute viable alternatives to it).[12]

A fortiori, the story of mathematical practices that obviously did *not* contribute to the development of Western mathematics, gets little to no

[11] Joseph (1987, 1997) uses the notion 'Europe' for all regions that are being dominated by a population of European origin, such as the United States, Canada, Australia, New Zealand etc. Powell and Frankenstein (1997) use the phrase 'Europe and Europeanized areas' in this respect, which is clearly is no longer restricted to the European continent.

[12] Cf., e.g., Grattan-Guinness (1994, part one).

mention in the classical history books. This is for example the case for ancient cultures like the Inca, or those from Sub-Sahara Africa. Whenever they are referred to, immediately the label 'ethnomathematics' is attached to it, as an immunizing move as it were. With respect to the historiography of mathematics the obvious alternative would be to acknowledge that more than one history of mathematics can be made up. A parallel, less geographically dominated dimension of the prevailing history of mathematics, has been the pushing aside of the 'gender' dimension. It has been ages since one thought that histories could be described that in principle incorporate the story of every human being. This neglect of *herstory* next to history might very well connect to the 'great men'-syndrome that has been so typical for much historiography of science.[13]

Summarizing, in general, mathematics has been historically pictured as an almost exclusive product of European society, at the cradle of which stood ancient Greece. Although it should be added that there is nowadays, among historians, a growing tendency of subtly modifying this harsh picture, next to that, there is also the issue of past and present mathematical systems, or rather collections of ideas, "that did not feed into or effect this main mathematical stream" (Ascher, 2002, p. 1). The reason is they were developed by peripheral and minor indigenous people. In paying attention to voices unheard in the Western dominated mathematical debate, the anthropological branch of ethnomathematics "has the goal of broadening the history of mathematics to one that has a multicultural, global perspective" (Ascher, 1998, p. 188), so as to include the unknown or misunderstood.

3.2.3 Philosophy

Drawing heavily upon "the theories, knowledge, and methods of culture history, cognitive studies and, above all, linguistics and anthropology" (Ascher and Ascher, 1994, p. 1545), ethnomathematics research may very well be subsumed under social studies of mathematical practices; practices which, importantly, are *not* all of a kind. In studies of science in general, the introduction of a cultural category has made it possible to more firmly establish the epistemic connection between (external) context and (internal) content. Important consequences, in history as well as in philosophy, have included a partial drive away from the traditional focus on individuals (the 'great men' syndrome), and the acknowledgement, as Steve Woolgar put it, "that reason, logic and rules are *post hoc* rationalizations of scientific and mathematical practices, not their determining force" (Woolgar, 1993, p. 50). What could then be the consequences of this type of research for the philosophy of mathematics? Could it come to challenge the dominant views that mathematical truth is immutable, monolithic, universal, timeless?

[13] Cf. also Koblitz (1996).

In his plenary lecture as an invited speaker at the 1950 International Congress of Mathematicians in Cambridge Mass., Raymond Wilder observed that his fellows had been invariably led, at least to a large extent, by what was considered or supposed to be proper mathematics in their own culture, and that particularly in the history of mathematics (of which he was an amateur himself) this limitation ought no longer to be overlooked (Wilder, 1998). Below, in §4.2, we shall show how Wilder himself, near the end of his life, actually proposed a philosophical endorsement of this view.

It can be no surprise that the feeling expressed by Wilder is widely shared by (mathematical) anthropologists, those in the business of studying other cultures, usually from within a relativist framework. According to them, alternatives to prevailing Eurocentric historiography should not just pay *more* attention to other traditions, past and present, but also, importantly, be *different* in character: maximally unbiased and detached, i.e., conducted in a 'true' anthropological spirit, taking 'the other' for granted, rather than (unconsciously) moulding aspects of it into fictitious categories that stand in outright opposition to our 'regular' or 'normal' Western ones (for example, 'the exotic').[14] What will be of interest at the philosophical level is not so much these empirical studies as such, but the way they affect our image of (the nature of) mathematics, Western and non-Western alike. Below, in §4.1, we briefly look into an approach called ethnomethodology, an application of a similar 'unbiased' perspective to the mathematics of the Western tribe itself, as exemplified in the work of Eric Livingston. After that, in §4.2, we turn to Wilder's own epistemological interpretation.

3.2.4 Education

A fourth and final field suited for ethnomathematical approaches is education. Here, all previous subdisciplines converge. For 'peripheral' countries, the critical historical and philosophical tradition with regard to superior Western mathematics accommodates a favouring of local mathematical practices in the curriculum, rather than (implicitly) importing the Western one. To that purpose, a description of everyday mathematical practices is needed, a task which anthropologists can take to heart. In response to the challenge of incorporating living mathematics into curricula and school practices, D'Ambrosio has formulated a programme of 'learning in action' (cf. Figure 1), in which the teacher's part is that of a process manager.

In view of the set of instruments and the social context, with content being delivered from a variety of sources, obviously a highly interactive approach is required. Let us however point out that D'Ambrosio is still overlooking one particular and very obvious source: the pupil. The general

[14] Cf., e.g., Said (1979) for a renowned complaint against such (implicit) methods, as applied to 'the orient'.

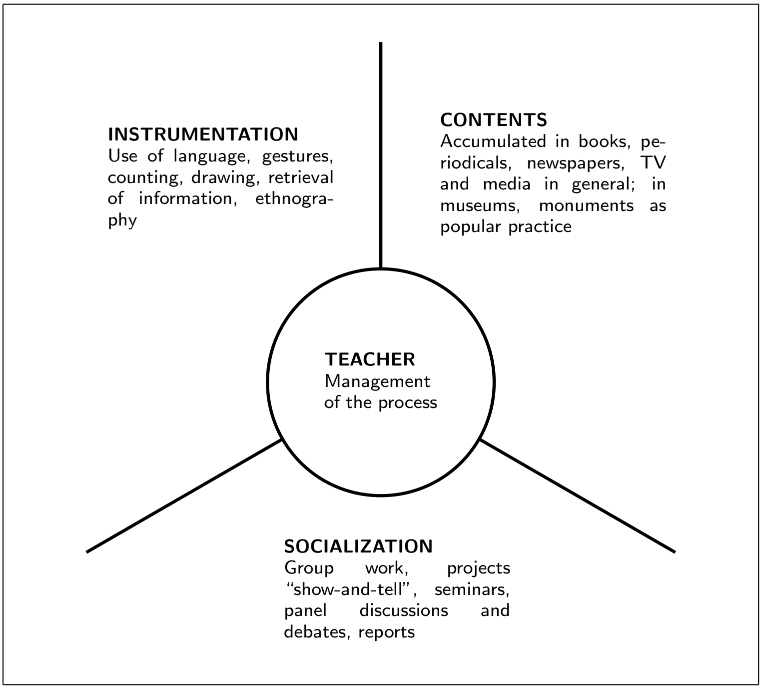

FIGURE 1. Interactive curriculum concept (D'Ambrosio, 1990, p. 376)

principle that the social environment of the person doing the learning must be taken into account provides a link between ethnomathematics and learning in diversity. In Section 5, we shall elaborate on the implementation of ethnomathematics in educational contexts.

4 Ethnomathematics and philosophy

4.1 Livingston

Among the more neutral definitions of ethnomathematics is one that refers to it as "the study of mathematical ideas and activities as embedded in their cultural context" (Gerdes, 2001, p. 12). As Paul Gerdes, the author of these words, himself indicates, this means that 'academic' mathematics is, in principle, also liable to the ethnomathematical approach. With his 1986 book *The ethnomethodological foundations of mathematics*, this is precisely what Eric Livingston has aspired to do: to 'descriptively analyse' what practices the "tribe of mathematical theorem-provers" are engaged in.[15] Livingston's

[15](Livingston, 1986). The source of the cited designation is (Livingston, 1999, p. 885).

central thesis is that of the local production of social order, in mathematics as in other scientific and non-scientific domains.[16] Social order in mathematics amounts to conveying and spreading a belief in the indubitable, 'transcendental' correctness of proofs. It is Livingston's contention that this is effectuated through, or supported by, nothing other than concrete displays of individual proofs (e.g., at the blackboard), despite –or rather thanks to– local contingencies. This approach has been criticized, by David Bloor among others, for being unable to explain how the processes described succeed in illuminating the nature of mathematical objects. Bloor contends that Livingston gets stuck in circularity between contingent practices and objective structures, as ethnomethodology "actually uses the very concepts whose questionable significance provided us with the philosophical problem with which we began. The whole problem was to illuminate what goes on when we 'realize' something in the course of a mathematical proof" (Bloor, 1987, p. 349). This way, says Bloor, Livingston does not in the least surpass the already sketchy accounts of both Lakatos and Wittgenstein, to which he owes substantial—though unacknowledged—debt. From Lakatos he takes, e.g., the idea that definitions are not laid down prior to the proof process, but emerge from that actual mathematical activity itself, or, more generally, that the phase of informal discovery intermingles with that of formal justification. With Wittgenstein, it shares the anti-foundational rationale for the primacy of concrete, low-level mathematical practice.[17]

It is fair to say that Livingston's ethnomethodology of mathematical proof remains theoretically very superfluous, resulting in a number of loose ends that are not attended to. From the circularity pointed out above flows a possibly unintended yet inescapable and strong sense of implicit Platonism, which does not square with its anthropological intentions.[18] There is another quasi-paradoxical point, in that the approach remains extremely

[16]Cf. the definition of Michael Lynch, where the approach to science in general is set out: "Ethnomethodology is commonly said to be the study of 'micro' social phenomena - the range of 'small' face-to-face interactions taking place on street corners and in families, shops, and offices" (Lynch, 1997, p. xii).

[17]Cf. "I go through the proof and then accept its result. – I mean: this is simply what we *do*. This is use and custom among us, or a fact of our natural history" (Wittgenstein, 1956, p. 61, i.e., RFM I–63).

[18]Cf. "Provers see through the representations of mathematical objects to the objects that are represented, and use those representations to inspect and discover properties of mathematical objects" (Livingston, 1999, p. 873). It should be added that in the paper just quoted from, it is additionally commented that ontological impartiality is implied here. However, something more seems to be needed in order to put into perspective persistent talk about mathematical objects and their representations. Although the bringing up of pebbles and diagrams seems to suggest that something like a (minimal) empiricist foundation of mathematical knowledge might be thought of here, these are not explicitly awarded more than illustrative power, and no (psychological) account is given of the passing from concrete to abstract practices (contrary to, e.g., Kitcher, 1983).

internalistic, with no attention paid to any of the small- or large-scale cultural surroundings (e.g., intellectual tradition) of the proofs 'lived through'. In sum, Livingston presents a genealogy of mathematical rigour that remains limited to a purely descriptive analysis of narrowly local practices. He takes us through specific proofs, e.g., by Euclid or Gödel, to show how they are supposed to convince (make certainty emerge). But the question of what actually constitutes their truth or falsity is *de facto* considered as totally irrelevant, and no normative criteria are proposed or (internal or external) explanatory forces identified. Therefore, let us now turn to a theoretically more elaborated anthropological account of mathematics, as Wilder attempted to give.

4.2 Wilder

When considering the body of original sources on 'humanistic' or 'non-foundational' approaches to mathematics (say, from the postwar era through the early 1980s), what is most striking is that nearly all of it appears to be by trained mathematicians, not philosophers. Raymond Wilder is exemplary of this tendency. As announced above, addressing the International Congress of Mathematicians as early as 1950, he urged his colleagues "to get outside mathematics, as it were, in the hope of attaining a new perspective, [...] [a] vantage point from which one can view such matters more dispassionately. [...] We 'civilized' people rarely think of how much we are dominated by our cultures—we take so much of our behavior as 'natural'" (Wilder, 1998, p. 186–187). One can hardly think of any philosopher of mathematics who would have (dared) put forward this type of externalist claim at the time. A full three decades later, in 1981, Wilder conceived the monograph *Mathematics as a cultural system*, in which he set out to systematically do what was only sketchily suggested in his earlier lecture: apply the anthropological approach to mathematics as a (Western) field of scientific inquiry. The following quote may function as a declaration of his programme.

> With the achievement of cultural status, such internal forces as hereditary stress, consolidation, selection, symbolization and abstraction have played an increasingly important part in the evolution of mathematical thought. To ignore these forces and the manner in which they have influenced characteristic patterns of evolution seems to deprive both the historian and the students who follow him of a fuller understanding of the historical process. Of course, mathematics is done by individuals, but these individuals share a common, albeit variable and diverse, mathematical culture, and along with the study of the achievements of a Gauss or Riemann, one should study the culture, both mathematical and environmental, in which they lived and worked, in order to achieve a fuller understanding and appreciation of what they accomplish. (Wilder, 1981, p. 162).

At the source of Wilder's work in the cultural anthropology of mathematics stands a general neglect—if not disdain—among mathematicians and philosophers alike for contemporary advances in the social sciences. Contrary to popular belief, the author puts forward the moderately naturalist thesis that mathematicians will greatly benefit, in terms of the deliberate choices they make between different problems and strategies, from considering and appreciating the cultural determinants of their work.[19] Wilder's approach is in fact adapted from that of Leslie White (1947), taking a cultural system as resulting from a dynamical interplay of 'vectorial' forces. In the case of mathematics, these forces are specified by Wilder as different types of 'stress' imposed on mathematical practices (see below). These influences vary, and the historical sequence of their different configurations constitutes mathematical evolution (indeed a largely neglected topic). Actually, this brings out specific affinities between Wilder's account and the operational part of Kitcher's approach, which we have elaborated on elsewhere (Van Bendegem and Van Kerkhove, 2004).

At a macro-level, a notable historical constant is that of the co-evolution (rise and decline) of mathematics and general culture (a factor also stressed by Marxists like Dirk Struik). For example, although the patterns that have followed the different specialties are very hard to discern, one may observe a general mathematical decline after the Greeks, and a subsequent resurrection, past the 'dark ages', from the seventeenth century onwards. This phenomenon Wilder calls *environmental stress.* "The environment suggested the invention of new concepts in mathematics, whose study resulted in mature techniques which were seized upon by environmental interests for the solution of their problems and advancement of their own theories" (Wilder, 1981, p. 55).[20] On top of this, note that developments have been regularly obeying the law of inertia. That is, even when in urgent need of accommodating present needs and worries, the time has to be ripe for launching new and possibly revolutionary ideas. For example, Saccheri failed to officially join the club of simultaneous developers of non-Euclidean geometry largely because of ideological reasons, since he was unable to free himself of the dominant notion of 'absolute' (physical or Platonic) truth. Similarly, Gauss felt way ahead of his time when it came to this topic, and for genuine fear of unfavourable public reaction left his work on it unpublished.

A form of cultural influence more limited in scope is *hereditary stress.* Individual mathematicians inherit a reigning culture of specific concepts and tools from their predecessors, and from within that context try to ac-

[19] A metamathematical example of such potential influence, suggested by Wilder himself, is that exerted by the development (also in other cultures!) of alternative logics, upon how to conceive of logicism.

[20] There is of course a strong link with the issue of applicability here.

complish set goals and contribute to the further development of their field and of mathematics generally. Wilder considers the following components connected with the mathematical domains scholars happen to be engaged in, and thus with the hereditary stress exerted on one freshly 'raised' in it: capacity, significance, challenge, status, and body of concepts (particularly its consistency). *Capacity*, or what remains to be done, while essentially indeterminate, is pretty easy for the trained mathematician to (roughly) assess. "If the capacity is large, he may either be attracted to do research in it, or direct others, such as younger colleagues or students, to consider helping to develop the field" (Wilder, 1981, p. 69). A domain with ample capacity is thus likely to attract talented people. And this, in its turn, might be a good way, although no guarantee, to reinforcing its further blossoming. A good example hereof is twentieth century Cantorian set theory, a counter-example the slow or even non-start of projective geometry in the seventeenth century. The element of *significance* is closely tied to that of capacity. Take computability theory, particularly in the light of the ever growing importance of the computer, for mathematics as for society in general. Exemplifying the move in the reverse direction is Euclidean geometry, or even geometry in general, having been increasingly marginalized during the modern process of rigorization and formalization. Also notice that these shifts are not necessarily 'internal' to mathematics, as is illustrated by the example of computer science, but also, e.g., that of additional fields connected with the military (such as cryptology or ballistics).

A third factor of hereditary stress, in its turn rather intimately connected with the first and second, is *challenge*, or the degree of difficulty of problems facing mathematicians. Assessing this dimension, one can refer to Hilbert-like commented lists of outstanding problems.[21] One of the most intriguing challenges recently brought to a good end was no doubt Andrew Wiles's feat, solving Fermat's Last Theorem. Famous challenging problems still available as we write this include Goldbach's Conjecture and the Riemann Hypothesis. A fourth element, again of a considerable cultural dependence, is that of the present *status* of the field in question. It is based on philosophical esteem (e.g., on whether the analytic or synthetic method is dominant),[22] but also on the services paid by mathematics to other fields in which it is applied, inside and outside science.

One of the regularities or patterns that marks the history of mathematics is that of 'multiples' or (quasi-)simultaneous discoveries, such as those of the calculus, non-Euclidean geometries or informational complex-

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[21] A couple of recent such lists can be found in Smale (1998); Griffiths (2000); Devlin (2002).

[22] Cf. (Otte and Panza, 1997).

ity theory.[23] This phenomenon, Wilder conjectures, is the uncommon result of severe *conceptual stress* (the large fifth hereditary category) exerted on mathematicians to avoid or resolve theoretical tensions, whether alleged or resulting from truly paradoxical situations. Threats to the internal consistency of mathematics boosts development, it is claimed, because they pose a special challenge. It is especially paradoxes connected with the infinite that have served this purpose very well: incommensurables and Zeno paradoxes in the Greek period; the continuum, Cantorian set theory, and the Russell antinomy in modern times. In the face of these, given culturally accepted axiom systems and proof methods, as well as uniform distribution of mathematical capacities, it is not so surprising that (quasi-)identical theorems are derived around the same time by different scholars. In general, conceptual stress is defined as the urge to devise a satisfactory conceptual system with suitable and properly interpreted symbolism. More examples of conceptual stress include the conception of the zeroth digit, of the imaginary numbers, or of the completed infinity as a number (by Cantor). All of these instruments, while workable, nevertheless did force a whole process of conceptual clarification (as did the Leibnizian ε-δ-symbolism for the calculus). But work on (non-paradoxical) specific problems might also require and thus induce conceptual innovation. E.g., solving general algebraic equations required group and Galois theory, and the foundations of analysis required set theory. An additional motive for conceptual innovation is the creation of order or unification. Examples include the case of Klein, whose "analysis of existing geometries led him to realization of the importance of transformation groups and geometrical invariants as their distinguishing features" (Wilder, 1981, p. 76), which supposedly brought order in the field of geometry, the introduction and gradual development of the group concept, and that of category theory.

In view of our topic of socio-history, it is interesting to eventually see Wilder himself propose, like Crowe, a number of developmental 'laws' for mathematics (cf. François and Van Bendegem, 2010, in this volume). These capture most of the dimensions covered and elaborated in the rest of his book in an explicit developmental pattern, inspired by and showing a substantial overlap with Crowe's laws, as there are: the role of hereditary, environmental and conceptual stress (and the interpretative flexibility in the face of the latter), the ubiquity of multiples, the decisive influence of status and beneficiary cultural climate, the limitations imposed by prevailing culture, the importance of tacit aspects or hidden assumptions (to be unearthed and made explicit), the cultural relativism and ultimately intu-

[23] A general sociological treatment of this phenomenon, at times—though in mathematics rarely—followed by harsh priority disputes, can be found in (Merton, 1973, §16 and §17). For a specific such analysis of the second case, cf. (Restivo, 1983, app. D).

itive basis of mathematical knowledge, diffusion between (sub)cultures, the driving force of conceptual fruitfulness (problem-solving capacity and aesthetic appeal), the absence of revolutions in the core of mathematics (cf. François and Van Bendegem, 2010, in this volume), the ongoing increase in rigour and abstraction, the occasional (and often erroneous) feeling that entire fields have been 'worked out'.

The whole of Wilder's 'anthropological' account might then be summarized thus: cultural evolution of mathematics follows its own laws, depending on various specific types of environmental (external) and—especially—hereditary (internal) stress, and as a process of change certainly has to be distinguished from the mere chronology of contingent historical facts.

5 Ethnomathematics and education

With D'Ambrosio as its intellectual father, the application of an ethnomathematical approach to education has Brazilian roots, but has since become common practice all over the world. In 1975, D'Ambrosio founded a masters programme in Teaching Sciences and Mathematics at the State University of Campinas (UNICAMP), to which, as we write, he is still affiliated as an emeritus. By doing this, and through taking part in the International Congresses on Mathematical Education (ICME), he established international contacts with other pioneers of mathematics education, such as Luis Santaló, Hans Freudenthal and Edward G. Begle. At the 1976 ICME-3 congress in Karlsruhe, Germany, participating in the panel discussing *Why teach mathematics?*, he was one of the first to assert that mathematics education is nested in a socio-cultural context. Since 1984, after the first explicit international articulation of his ideas regarding ethnomathematics during his plenary lecture "Socio-Cultural Bases for Mathematical Education" at the fifth ICME conference in Adelaide, Australia, D'Ambrosio has developed his ideas in full detail through numerous articles. In 1985, he was the co-founder of the International Study Group on Ethnomathematics (ISGEm).

Meanwhile ethnomathematics has developed into both an education practice and a separate discipline within academics. From the start, D'Ambrosio has emphasized that with the notion 'ethnomathematics' he aspired to cover a much larger area than in its traditionally ethnocentric or exotic interpretation, and that the approach should be applicable to any learning context. He holds on to this distinction through what he calls ethnic-mathematics as opposed to ethnomathematics (D'Ambrosio, 2007a, p. 30).

Even though ethnomathematics is a critical research program and practice regarding mathematics education, it should still be considered different from so-called Critical Mathematics Education (Vithal and Skovsmose, 1997, pp. 132–133). That is, although both research programs reside within the much larger Learning in Diversity research community, ethnomathematics originated in post-colonies that have opposed themselves to importing a Western curriculum, instead developing their own mathematical practices that could serve as an instructive basis for education; while Critical Mathematics Education originated from within Western high-tech societies criticizing the idea of linear progress, and defining a number of suppression types, for example those based on class and gender (Atweh et al., 2001; Burton, 2003).

Within ethnomathematical studies, one can distinguish two major, but interrelated points of attention. On the one hand, as an application of human and childrens' rights in general, the importance of mathematical education and literacy for all, that is, in all cultures and for each and every one within those cultures. This is the theme of §5.2 and §5.3. On the other hand, the diversity of mathematical practices across cultures, a diversity which, in order to challenge and supersede a monolithical and universalist image of mathematics, should be represented in the math curricula. This is the theme of §5.1. As said, both themes are connected. Culturally adjusted math education does no longer import Western curricula without further ado, but actively examines and uses local practices as an entry to the subject matter. As a result, both a better access to math education and a recognition of mathematical diversity are served.

5.1 Cultural diversity

As already explained above (§3.1), the meaning of the 'ethno' concept has been extended since the beginning of its application, now designating ethnical groups, national groups, racial groups, professional groups, groups with a philosophical or ideological basis, socio-cultural groups and groups that are based on gender or sexual identity respectively (Powell, 2002, p. 19). Although this list is in principle incomplete, and some distinctions are or have been relevant only in specific contexts, it is clear that throughout the history of the concept 'ethno', elements of diversity have always been aimed at. Accordingly, with respect to the field of mathematics, 'ethnomathematics' can be characterized as being engaged with cultural diversity within the whole of mathematics. Comparative cultural studies of mathematics, while describing various mathematical practices, not only reveal their diversity, but also emphasize the complexity of each system, by showing how these practices arise and are used within specific socio-cultural and historical contexts. Finally, this approach is translated to mathematics education, challenging the instructor to introduce a diversity of mathematical practices in the curriculum.

The latter application exceeds the mere introduction in class of the study of other cultures or—to put it more dynamically—new culture fields. Indeed, this was done at first, namely looking into practices related to mathematics (and adjacent sciences) as part of the enquiry of all kinds of exotic traditions: music from Brazil, card games played in Madagascar, the arithmetic system of the Incas or the Egyptians, basket or carpet weaving, the Mayan calendar, the production of dyes out of natural substances, tea drinking habits or terrace cultivation in China, water collection in the Kalahari desert, the construction of Indian arrows, house construction or sand drawing in Africa, etc. (Bazin et al., 2002). Notwithstanding the good intentions of these and similar projects, we would like to emphasize, with reference to (Powell and Frankenstein, 1997), that these initiatives, while originally intended to offer intercultural education, run an easy risk of turning into some kind of folklore. It should be clear that we ourselves are not advocating such a simplistic curricular use of other people's ethnomathematical knowledge. Quite to the contrary, many examples show that foreign mathematical practices may be used in class not as some kind of exotic diversion next to 'the real stuff', but as truly mathematical in their own right. For instance, interesting cases about the number concept or certain geometrical aspects have been elaborated (Zaslavsky, 1985; Katz, 1994). Now pupils not only learn about how people elsewhere count or have originated the decimal, vigesimal or duodecimal systems; they also get to know how the various mathematical practices were (and continue to be) answers to the circumstances in which they arose.

This is of more than local importance. For example, while most European languages use the decimal system, they still make lots of references to the vigesimal (e.g., "quatre-vingts" in French) and the duodecimal systems (e.g., "half a dozen of eggs", or the watch), systems which are closely related to the structure of the human body: fingers, toes, finger bones, … In Sierra Leone's *Mena* language, twenty stands for a complete person, which derives from groups of twenty representing all fingers and toes. Counting all finger bones (excluding the thumb, which is the instrument used to count with) results in the figure of twelve. Taking the fingers of the other hand to register the number of times that we reach twelve with the one, the result will be 5 times 12 or 60. Another example pertains to the geometrical concepts of area, perimeter and their ratio. Based on the construction of houses all over the world, students try to work out the most economical form of building when materials are restricted (Zaslavsky, 1985, 1989; Pinxten, 1994). By comparing the ratio between area and perimeter of various surfaces (circle, square, rectangle, triangle, …), students should conclude circular constructions are most economical.

In (Lengnink and Schlimm, 2010, in this volume), an additional illustration is provided, analyzing the common difficulties in learning arithmetic in the decimal place-value system, and discussing pedagogical strategies for overcoming the difficulties. By comparing the algorithms for operations in the decimal place-value system with the additive, Roman numeral system, they have been able to show that lots of the mistakes students make when learning the decimal place-value system would not occur within the additive system. This analysis of the semantic aspects of number representation is an appropriate example of how techniques other than the common mathematical operations may be successfully used in class.

Let us give one more example, viz. about ethnocomputing. Indeed, ethnomathematics has recently evolved so dramatically that even digital applications now exist. It was actually but a small step for Eglash et al. (2006) to develop a software package that places mathematical ideas and practices in a cultural context. The Culturally Situated Design Tools (CSDTs) software more particularly gives students the possibility of reproducing art of different cultures, ranging from Indian strings of beads or Afro-American hair braiding to metropolitan graffiti. This can be done in an independent and creative way, exploiting underlying mathematical principles. With their program, the designers explicitly want to counter multicultural mathematics, which they regard as mostly holding an exotic as well as a static vision on culture. Their contrasting ethnomathematical view is based on four principles. First, mathematical designs are by no means autonomous, but are linked to other cultural expressions such as cosmology, spirituality, medicine,... Second, the complexity of mathematical designs refutes their alleged primitiveness. Third, not only are the designs interpreted from a Western perspective, for example looking for symmetries, but they are also analyzed with an eye on discovering underlying models that reappear in other designs. Fourth, they herald a dynamical view of culture.

In the following section, we look into the philosophy of egalitarianism and emancipation that helped bring about the changes that allowed for the implementation of the classroom practices just discussed.

5.2 Egalitarianism

The ethical-theoretical foundations on which egalitarian projects within education are based, assume that equality is measured at the end of the educational process. For as pointed out in the justice theories of Rawls (1999) and Sen (1992), pupils' starting positions can be dissimilar in such a way that strictly equal deals will prove insufficient to achieve equality. This implies that a meritocratic approach—which measures equality at the start of the process—cannot fully guarantee equal chances. Egalitarian approaches, therefore, start from a certain pedagogic optimism, taking into account the diversity of those learning in order to give them the greatest chance of equality.

By extending the notion 'ethno' to include diversity, it has been claimed, the distinction between mathematics and ethnomathematics might be disappearing. Hence the question has been raised whether thereby the achievements of ethnomathematics will not also get lost. We think not, as the said distinction can and will only disappear once ethnomathematics' achievements have been acknowledged and implemented in mathematics education (Pinxten, 1994; Setati, 2002). Nevertheless, although ethnomathematics was originally mostly just critical of (the dominance of) Western mathematics, now the time seems to have come to also raise critical questions in and about the programme itself. What indeed exactly distinguishes ethnomathematics from mathematics, if all of mathematics is constituted by mathematical practices performed by cultural groups that identify themselves on the basis of philosophical and ideological perspectives (Setati, 2002, p. 31)?

To make this point more specific with respect to education: Every maths teacher is supposed to apply a series of standards that come with the profession. These standards are philosophical, ideological, and argumentative in nature, pertaining to ways of being, ways of perceiving and ways of expressing respectively. Hence both mathematics and ethnomathematics turn out to be embedded in a normative framework, which casts doubts over whether the values of mathematics and ethnomathematics are really that distinctive.

What does distinguish ethnomathematics as a programme, is its emancipatory and critical spirit, promoting the equality of groups hitherto discriminated against (Powell and Frankenstein, 1997). To use Gilligan's (1982) terms, ethnomathematics gives air to the *different voices* in mathematics education. Carrying this objective into education in general, institutions organizing education mostly do so from an emancipatory perspective. This very egalitarian idea of respecting diversity is also to be found in UNESCO's educational mission. This connects mathematics education closely with wealth, endurance and peace, to converge in a general human rights discourse. Indeed, UNESCO firmly believes that education is the key to sustainable social and economic development, as the following quote from the UNESCO mission statement shows:

> The mission of the UNESCO Education Sector is to: Provide international leadership for creating learning societies with educational opportunities for all populations; Provide expertise and foster partnerships to strengthen national educational leadership and the capacity of countries to offer quality education for all.

Having established that general educational objectives clearly relevant to the issue of equal opportunities, which is central to the ethnomathematics programme, let us now turn to the philosophical question of the value-ladenness of mathematics education.

5.3 Values and equal opportunities

D'Ambrosio has situated mathematics education within a social, cultural and historical context, and also explicitly linked mathematics education with politics (D'Ambrosio, 1997). According to him, mathematics education is a lever for the development of the individual, national and global well-being (D'Ambrosio, 2007a,b). In other words, teaching mathematics has a political foundation. D'Ambrosio has advanced that mathematics education should hence be available to all; a proposition which has been registered in the OECD/PISA report, the basis of the PISA-2003 continuation enquiry.

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (Watanabe and McGaw, 2004, p. 37)

This specification of mathematical literacy clearly implies that it has now explicitly been acknowledged as a basic right for every child, allowing him or her to participate in society in a constructive, relevant and thoughtful way. The theme recurs in the essays of Alan Bishop, demonstrating the link between mathematics, ethnomathematics, values and politics (cf., e.g., Bishop, 2002; Bishop et al., 2006). Because mathematics is traditionally perceived as being non-normative *par excellence*, the mere mentioning of mathematics education and education of values in the same breath sounds ambiguous. However Bishop comments:

> It is a widespread misunderstanding that mathematics is the most value-free of all school subjects, not just among teachers but also among parents, university mathematicians and employers. In reality, mathematics is just as much human and cultural knowledge as any other field of knowledge, teachers inevitably teach values... (Bishop, 2002, p. 228)

Be that as it may, mathematics education is a field in which a multicultural approach, dealing with diversity or cooperative learning is very hard to find. Indeed, mathematics does not appear to easily lend itself to integrating the so-called 'cross-curricular' learning objectives, such as consumer, health, environmental, safety or leisure education. Bishop (1997) observes that the curricula, on the contrary, have a particularly narrow, technical orientation, in which there is (almost) no room for historical considerations or philosophical exercise. We have managed to confirm this on the basis of our own research into the role of philosophy in the mathematics curriculum of secondary education in Flanders–Belgium (François and Van Bendegem, 2007).

Bishop was one of the first to investigate values in relation to mathematics and mathematics education (Bishop, 1997). To facilitate their study, he identified a cluster of six values, presumably used by mathematics and science teachers (Bishop et al., 2006). There are three groups of two values each. The first group touches on the ideological dimension: rationalism and objectivism. The second group connects with the attitudinal dimension: control (prediction, checks and rules) and progress (cumulative knowledge). The third group introduces the sociological dimension: openness (knowledge as a human activity and product) and mystery (the 'unreasonable' origins and uses of knowledge).

In line with Bishop, Paul Ernest also focuses on the analysis of values in the context of mathematics and mathematics education. He holds that philosophy of mathematics is currently at the heart of a Kuhnian revolution. As he has it, mathematics has been dominated by an absolutist paradigm for more than two thousand years: it has been considered to be a foolproof system consisting of objective truth, hence steering clear of values. In the past decades, however, the viability of studying mathematics just as any other branch of human knowledge has become increasingly obvious. In this spirit, where one does not exclude fallibility, change or cultural context, mathematics is conceived as a knowledge system that is the result of human investments. Ernest (1991) has analyzed different ideological groups and the values that they adhere to within mathematics education. As for the inherent values of mathematics, he has documented that 'abstract' is esteemed higher than 'concrete', 'formal' higher than 'informal', 'objective' higher than 'subjective', 'justification' higher than 'discovery', 'rationality' over 'intuition', 'reason' over 'emotion', 'general' over 'particular', 'theory' over 'practice', 'intellectual' over 'manual', etc. (Ernest, 1991, p. 259). Prediger (2006) has added some more values to this list, by showing that 'coherence' and 'consensus' are held in particularly great respect by the community of mathematicians, and that big efforts are made to remove instances of inconsistency or dissensus, whenever they occur.

It is particularly within ethnomathematics research that the link between mathematics education and values is further extended into the political domain. According to D'Ambrosio, too many people are still convinced that mathematics education and politics have nothing in common. In recent work (D'Ambrosio, 2007a,b), he brings to attention the Universal Declaration of Human Rights, articles 26 and 27 of which highlight the right to education and the right to share in scientific advancements and their benefits. These were further developed and confirmed in UNESCO's World Declaration on Education for All (1990), ratified by 155 countries. The principles have also been applied in the OECD/PISA declaration on mathematical literacy (2003). D'Ambrosio deeply regrets that these manifestos are not at all well-known by maths teachers, given that it is precisely they who play a key role in the emancipatory process at which the declarations aim.

To stress the actual importance of the matter, D'Ambrosio once more points out that mathematics is at the basis of many significant developments in technology, industry, the military, economy and politics, fields in which many problems faced by mankind are embedded. D'Ambrosio then argues that mathematics and its education are valuable tools to help solve these problems. At this point, he attributes a key role to the study of mathematical practices showing mathematics to be a creative force.

> I see my role as an educator and my discipline, mathematics, as complementary instruments to fulfill commitments to mankind. To make good use of these instruments, I must master them, but also need to have a critical view of their potentialities and of the risk involved in misusing them. This is my professional commitment. (D'Ambrosio, 2007a, p. 27)

According to D'Ambrosio, a critical way of teaching mathematics does not get enough chances. Like Bishop (1997) he criticizes predominantly formally-oriented curricula with their emphasis on technical drill, giving hardly any place to history, philosophy or reflection in general. D'Ambrosio has proposed three concepts to focus on in a new curriculum, if one wants to take the international (UNESCO) emancipatory objectives to heart: literacy, matheracy and technoracy. Literacy pertains to communicative skills in containing and using information. Both spoken and written language are concerned, but so are symbols, meanings, codes and numbers, so mathematical literacy is undoubtedly part of it. Matheracy denotes the necessary qualities of a scientific attitude: being capable of developing hypotheses, deducing and drawing conclusions from data. Technoracy refers to opportunities to familiarize oneself with technology, such that every pupil at least has the chance to get to know the basic principles, possibilities and risks of technological artifacts.

To conclude this section, let us summarize that the ethnomathematics programme is a value-driven view on mathematical practice and education. In sharp contrast to most Western mathematical research and educational traditions, in ethnomathematics practices are not decoupled from their cultural origins and contexts. More strongly than that, education is explicitly associated with social justice and even politics.

6 Conclusion

In this paper, we have aimed to give a general overview of how the field of ethnomathematics has developed from its origins in anthropological studies of mathematical practices outside the 'Western' tradition. It has been observed that, because of their implicit or explicit criticism of Eurocentrism, these studies have inspired a growing interest in the cultural embedding of mathematical practices, including 'Western' ones. Within the philosophy of mathematics, this has added to the array of studies, arising since the middle of the twentieth century, that collectively propose themselves as 'alternative' to foundational studies, by bringing back the nature of mathematical knowledge to where it seems to belong: in the practices of concrete, limited and fallible human beings.

With respect to mathematics education, a number of crucial evolutions should be noted as well. Less developed countries have increasingly ceased merely to import the 'superior' Western mathematical curricula, and instead have begun implementing local practices in education. In the developed world itself, the interest has moved (or widened) from exotism to cultural diversity: Ethnomathematics in school no longer means a five minute break from technical drill, but has become one of the instruments for better learning to deal with cross-cultural differences within or without the immediate environment. The philosophies of egalitarianism and emancipation have influenced these changes in education, to the point that they have also been written into chapters of global political programs (UNESCO, OECD, PISA) that strive to ensure every human's right to mathematical literacy.

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